Possibility to determine the astrophysical S factor for the ^{7}Be$(p,\gamma)^8$B radiative capture from analysis of the ^{7}Be$(^3$He,$d)^8$B reaction

A. M. Mukhamedzhanov and R. E. Tribble
Cyclotron Institute, Texas A&M University, College Station, Texas 77843

N. K. Timofeyuk
Institute of Nuclear Physics, Uzbek Academy of Sciences, Ulugbek, Tashkent 702132, Uzbekistan
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At very low energies of astrophysical interest, the ^{7}Be$(p,\gamma)^8$B reaction is almost totally peripheral. Consequently, the overall normalization of the cross section for this reaction is defined by the asymptotic normalization coefficient of the bound-state wave function of ^{8}B in the two-body channel ^{7}Be+p. The reaction ^{7}Be$(^3$He$,d)^8$B is suggested as a tool to measure this asymptotic normalization coefficient thereby allowing one to define the absolute value of the astrophysical factor, $S_{17}(0)$, for the ^{7}Be$(p,\gamma)^8$B reaction.

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I. INTRODUCTION

The ^{7}Be$(p,\gamma)^8$B reaction occurs in one of the weakest of the three p-p chains in the hydrogen burning of main-sequence stars. Even so, its rate at very small energies ($E_{\rm c.m.} < 25$ keV where $E_{\rm c.m.}$ is the relative kinetic energy of p and ^{7}Be in the entrance channel) is of fundamental importance, both for defining the branching ratios between the different p-p chains and for calculating the high-energy solar neutrino flux which is of special interest for the solar neutrino problem. At astrophysical energies, the cross section for this reaction is so small that its measurement has not been possible to date. Measurements have been carried out at energies $E_{\rm c.m.} > 117$ keV. The experimental cross sections have then been extrapolated to zero energy [1]. The overall uncertainty in the astrophysical factor $S_{17}(0)$ derived in such a procedure has been estimated to be about 30% [2]. Another possibility for finding $S_{17}(0)$ is to measure the Coulomb breakup of ^{8}B [3]. However, contributions from $E2$ excitation [4] and Coulomb post-acceleration may significantly distort the result extracted from this process for astrophysical applications, where only the $E1$ contribution is relevant.

Theoretical calculations have also attempted to determine this reaction rate at solar energies. Calculations have been made in the potential model [5–7] and in the microscopic generator-coordinate method (GCM) [8]. In the potential model, the overlap function (reduced width amplitude) of the internal bound-state wave functions of ^{8}B and ^{7}Be, $I_{^8{\rm B}\,^7{\rm Be}}(\mathbf{r}) = \langle \phi_{^7{\rm Be}} | \phi_{^8{\rm B}} \rangle$, where $\mathbf{r}$ is the relative coordinate between the proton and the center of mass of ^{7}Be, is approximated by $S^{1/2}_{^8{\rm B}\,^7{\rm Be}}\phi_{^7{\rm Be}p}(\mathbf{r})$. Here $S_{^8{\rm B}\,^7{\rm Be}}$ is the spectroscopic factor of the configuration ^{7}Be+p in ^{8}B and $\phi_{^7{\rm Be}p}(\mathbf{r})$ is the shell-model single-particle (proton) bound-state wave function in ^{8}B that is typically the solution of the single-particle Schrödinger equation with the phenomenological Woods-Saxon potential. However, in Ref. [7] the quasimicroscopic folding procedure has been used to find the shell-model single-particle potential. The overall normalization of the cross section of the peripheral radiative capture, ^{7}Be$(p,\gamma)^8$B, is defined then by the product $S_{^7{\rm Be}p}b^2_{^7{\rm Be}p}$, where $b_{^7{\rm Be}p}$ is the asymptotic normalization coefficient (ANC) of the tail of the single-particle bound-state wave function $\phi_{^7{\rm Be}p}(\mathbf{r})$, whose value depends on the geometrical parameters (radius r_0 and diffuseness a) of the adopted Woods-Saxon potential.

The peripheral nature of radiative capture reactions at very low energies and for small binding energies of the final nuclei is well understood [9]. Christy and Duck pointed out that in such reactions the overall normalization of the cross section is defined by the reduced width, which is expressed in terms of the product of the spectroscopic factor and the single-particle reduced width. However, the parametrization of the cross section in terms of the product of two quantities, the spectroscopic factor and the single-particle ANC, or the spectroscopic factor and the single-particle reduced width, each of which is model dependent, causes an uncertainty in the absolute normalization of the cross section. This has been explicitly demonstrated by Barker [5] who estimated the range of values for $S_{17}(0)$ to be between 0.012 and 0.020 keV b, based on uncertainties for the spectroscopic factor and geometrical parameters of the Woods-Saxon potential. The microscopic GCM is supposed to be a more accurate method for determining the reaction rate than the potential approach, but the overall normalization of the cross section is very sensitive to the adopted NN potential [10,11]. Previously, we have shown [12,13] that due to its peripheral character, the cross section for the ^{7}Be$(p,\gamma)^8$B reaction at $E_{\rm c.m.} \to 0$ is parametrized in terms of only one parameter, the ANC of the overlap function $I_{^8{\rm B}\,^7{\rm Be}}$. This ANC is related to $S_{^8{\rm B}\,^7{\rm Be}}$ and $b_{^7{\rm Be}p}$ by

$$C_{^8{\rm B}\,^7{\rm Be}} = S^{1/2}_{^8{\rm B}\,^7{\rm Be}}\, b_{^7{\rm Be}p}. \qquad (1)$$

Hence to calculate $S_{17}(0)$, it is sufficient to know the

value of the ANC's defining the overall normalization of the cross section (actually one needs to know two ANC's corresponding to the two ^{7}Be+p channel spin values: S = 1, 2).

The ANC is a fundamental nuclear characteristic, which plays an important role in nuclear structure and nuclear reaction theory [14,15]. The ANC for the virtual decay, $a \to b + c$, defines the probability of finding a nucleus a in the configuration $b + c$ at distances between b and c larger than the nuclear interaction radius of these particles. That is why the cross sections of peripheral radiative capture reactions are parametrized in terms of the ANC's rather than spectroscopic factors. We note that the spectroscopic factor of the configuration $b + c$ in nucleus a is, by definition, the norm of the overlap integral $\langle \phi_b \phi_c | \phi_a \rangle$ and is determined primarily by the behavior of the overlap function in the nuclear interior, which does not contribute to the peripheral radiative capture reaction $b + c \to a + \gamma$.

ANC's have been determined both by experimental data and theoretical calculations [14,15]. One of the reliable methods for obtaining ANC's is to extrapolate experimental data to the point of a nearest singularity in the E or $\cos\theta$ plane. For example, the ANC's governing the overall normalization of the S factor for the astrophysical reaction $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}+\gamma$ were found in [16] by the extrapolation of the S-wave α-^{3}He scattering phase shift to the pole in the E plane corresponding to the bound state of ^{7}Be. The same procedure has been used in Ref. [17] to define the ANC for $^6\mathrm{Li} \to \alpha + d$.

In previous publications [12,18], the ANC's of the overlap integral for the virtual decay $^8\mathrm{B} \to {}^7\mathrm{Be} + \overline{\overline{p}}$, for the channel spins $S = 1$ and $S = 2$, have been calculated in the microscopic approach developed in Ref. [18] and the result for the astrophysical factor was found to be $S_{17}(0) = 0.0165$ keV b. More accurate calculations of $S_{17}(0)$, using the same approach but taking into account S- and D-wave contributions in the initial state of the reaction $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$, gave $S_{17}(0) = 0.0176$ keV b [13]. This value turns out to be significantly smaller than the value $S_{17}(0) = 0.0225$ keV b used to calculate the high-energy ^{8}B neutrino flux in the standard solar model [19] and in the Turck-Chiéze *et al.* model [20], but it is within the interval found in Ref. [5]. The results of Refs. [12,13] depend completely on the accuracy of the calculated ANC's. We note that the ANC's for $^8\mathrm{B} \to {}^7\mathrm{Be}+p$ were calculated using a microscopic approach [18] with the NN potential M3YE (M3Y potential in the Elliott form) [21]. This potential was best able to reproduce (with fixed parameters) three well-established ANC's for $^7\mathrm{Li} \to {}^6\mathrm{Li}+n$, $^{13}\mathrm{C} \to {}^{12}\mathrm{C}+n$, and $^{14}\mathrm{N} \to {}^{13}\mathrm{N}+n$, which were found from the analysis of the experimental cross sections of reactions (p,d), (d,t) and heavy-ion-induced neutron transfer reactions [22–25]. The two first coefficients also have been derived by extrapolation of the experimental data to the pole singularity of the cross section in the $\cos\theta$ plane. This pole is the closest singularity to the physical region and corresponds to the neutron transfer mechanism [14,22]. The typical uncertainty for the phenomenological ANC's was about 10%. It is difficult to evaluate the uncertainty in the ANC's calculated in Ref. [18] since the theoretical values of the ANC's were derived using a fitting procedure for the theoretical calculations to the known phenomenological ANC's. This is why it is of great importance to find the experimental values of the ANC's for the virtual decay $^8\mathrm{B} \to {}^7\mathrm{Be}+p$ which define the overall normalization of the reaction $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ cross section at astrophysical energies.

Of course, one may try to extract the ANC's under consideration from the available low-energy experimental data but even the latest results derived by Fillipone *et al.* [1] are believed to have an uncertainty of about 30% [2]. We also note that the experimental data cover the energy range 117 keV$< E_{\mathrm{c.m.}}$. At such energies, the influence of the resonance at $E_{\mathrm{c.m.}} = 632$ keV is more significant than at zero energy. To find the $S_{17}(0)$ requires extrapolating the experimental data down to zero energy which can induce an additional uncertainty in its value. Thus independent measurements (direct or indirect) of $S_{17}(0)$, or equivalently, of the ANC's for $^8\mathrm{B} \to {}^7\mathrm{Be}+p$, are necessary. We propose in this paper a different method to extract the ANC's for $^8\mathrm{B} \to {}^7\mathrm{Be}+p$ using peripheral proton transfer reactions.

Recently, as a result of the development of techniques for the production of secondary beams, modern nuclear physics has a new tool to investigate processes involving radioactive nuclei. This possibility can be useful to define the ANC's under consideration. Using ^{7}Be or ^{8}B radioactive beams, it is possible to study transfer reactions at small angles, such as $a(^7\mathrm{Be}, {}^8\mathrm{B})b$ or $b(^8\mathrm{B}, {}^7\mathrm{Be})a$, where $a = b + p$, and choose a target and beam energy in such a way that the contribution from the interior part of the $I_{^8\mathrm{B}\,^7\mathrm{Be}}(\mathbf{r})$ would be negligible. Then the cross section for such a transfer reaction would be proportional to the sum of the squares of the ANC's, $C^2_{^8\mathrm{B}(1^+_{\frac{1}{2}})} + C^2_{^8\mathrm{B}(1^+_{\frac{3}{2}})}$ (see below). But just this sum defines the overall normalization of the astrophysical S_{17} factor at solar energies. This technique appears to us to be better than possible $p(^7\mathrm{Be},\gamma)^8\mathrm{B}$ measurements, since there is no need to perform measurements at very low energies where the cross section is extremely small, or to extrapolate the experimental data to zero energy to get the $S_{17}(0)$ value. The best way to determine the ANC's values from the $a(^7\mathrm{Be}, {}^8\mathrm{B})b$ reaction would be to use a heavy target a. Because of strong absorption, this reaction would be purely surface. But since the amplitude of such a reaction also contains the overlap integral of the internal wave functions of nuclei a and b, $\langle \phi_b | \phi_a \rangle$, the cross section of the peripheral reaction is proportional to the product of the squares of two ANC's, for $^8\mathrm{B} \to {}^7\mathrm{Be}+p$ and $a \to b+p$. Thus an accurate knowledge of the ANC of the overlap integral $\langle \phi_b | \phi_a \rangle$ is required. Unfortunately, the ANC's for one-proton removal are not known with good accuracy, and in the case of a heavy target a, one needs additional experiments to establish the ANC for the virtual breakup $a \to b + p$.

For purposes of illustration, we consider the possibility of determining the ANC's for the virtual breakup $^8\mathrm{B} \to {}^7\mathrm{Be}+p$ using a ^{3}He target, since the ANC for the $\langle \phi_b | \phi_a \rangle$ overlap integral, where $a = {}^3\mathrm{He}$ and $b = d$, is known with high accuracy [26]. To find the ANC for $^3\mathrm{He} \to d + p$, a few hundred precise experimental cross

sections (with overall uncertainties $< 2\%$) have been analyzed for the reaction $d + d \rightarrow t + p$ at 19 different incident energies in the interval $4.0 \leq E_d \leq 83$ MeV [26]. The analysis method was explained in Ref. [27]. By extrapolation of the experimental data in the $\cos\theta$ plane to the neutron transfer pole, which is nearest to the physical region singularity of the cross section, the triton vertex constant for $^3\mathrm{H} \rightarrow d + n$ in the channel with zero relative angular orbital momentum of d and n, and total angular momentum for the neutron of $j_{^3\mathrm{H}} = 1/2$, has been extracted: $G^2_{^3\mathrm{H}(0^+_{\frac{1}{2}})} = 1.34 \pm 0.02$ fm. The vertex constant for $^3\mathrm{He} \rightarrow d+p$, $G_{^3\mathrm{He}(0^+_{\frac{1}{2}})}$, should be very close to $G_{^3\mathrm{H}(0^+_{\frac{1}{2}})}$ since the increase of the vertex constant for ^{3}He due to the Coulomb interaction between d and p is compensated by the corresponding decrease of the vertex constant due to the lower binding energy between d and p compared to that of d and n. From the analysis of S-wave p-d and n-d scattering using the numerator/denominator or N/D equations, the difference between $G_{^3\mathrm{H}(0^+_{\frac{1}{2}})}$ and $G_{^3\mathrm{He}(0^+_{\frac{1}{2}})}$ has been found to be $|G_{^3\mathrm{H}(0^+_{\frac{1}{2}})}|^2 - |\tilde{G}_{^3\mathrm{He}(0^+_{\frac{1}{2}})}|^2 = 0.05$ fm [28], where $G_{^3\mathrm{He}(0^+_{\frac{1}{2}})} = e^{\frac{i\pi\eta_{^3\mathrm{He}}}{2}}\Gamma(1+\eta_{^3\mathrm{He}})\tilde{G}_{^3\mathrm{He}(0^+_{\frac{1}{2}})}$. Taking into account the relation between the vertex constant and ANC for the virtual decay $a \rightarrow b + p$,

$$G_{a(l_a j_a)} = -\frac{\sqrt{\pi}}{\mu_a} e^{\frac{i\pi(\eta_a + l_a)}{2}} C_{a(l_a j_a)}, \tag{2}$$

where all the quantities are defined below and the antisymmetrization factor allowing for the nucleon identity has been absorbed into the ANC. The ANC for $^3\mathrm{He} \rightarrow d + p$ in the channel with $l_a = 0$ is found to be $C^2_{^3\mathrm{He}(0^+_{\frac{1}{2}})} = 3.9 \pm 0.06$ fm^{-1}. We use a system of units such that $\hbar = c = 1$. When analyzing the data containing the vertex $^3\mathrm{He} \rightarrow d + p$, both components $l_a = 0$ and $l_a = 2$ should be taken into account. But for ^{3}He the ratio of the squares of the vertex constants (or ANC's) for $l_a = 0$ and $l_a = 2$ is about 5% [29]. Thus the contribution of the $l_a = 2$ component in the vertex for $^3\mathrm{He} \rightarrow d + p$ was neglected.

II. DISTORTED-WAVE BORN APPROXIMATION DIFFERENTIAL CROSS SECTION

To check our proposal, we investigated the peripheral character of the reaction

$$^7\mathrm{Be} + {}^3\mathrm{He} \rightarrow {}^8\mathrm{B} + d. \tag{3}$$

The cross section for this reaction was calculated within the framework of the distorted-wave Born approximation (DWBA). The DWBA amplitude for the reaction $X + a \rightarrow Y + b$, where $X = {}^7$Be, $Y = {}^8$B, $a = {}^3$He, $b = d$, is given by

$$M = \sum_{M_p} \langle \Psi_f^{(-)} I_{YX} \mid V_{bp} \mid I_{ab} \Psi_i^{(+)} \rangle. \tag{4}$$

Here V_{bp} is the interaction potential between particles b and p, which, for simplicity, is assumed to depend only on the relative coordinate $\mathbf{r}$ between the center of mass of b and p. The wave functions $\Psi_i^{(+)}$ and $\Psi_f^{(-)}$ are the distorted waves in the initial and final channels,

$$I_{\alpha\beta}(\mathbf{r}) = \sum_{l_\alpha m_\alpha, j_\alpha \nu_\alpha} i^{l_\alpha} (J_\beta M_\beta \; j_\alpha \nu_\alpha \mid J_\alpha M_\alpha) (J_p M_p \; l_\alpha m_\alpha \mid j_\alpha \nu_\alpha) I_{\alpha\beta l_\alpha j_\alpha}(r) Y_{l_\alpha m_\alpha}(\hat{\mathbf{r}}) \tag{5}$$

is the overlap integral of the internal bound-state wave functions of nuclei α and β ($\alpha = \beta + p$, $\beta = b, X$), where $I_{\alpha\beta l_\alpha j_\alpha}(r)$ is its radial part, $Y_{l_\alpha m_\alpha}(\hat{\mathbf{r}})$ are the spherical harmonics, $\hat{\mathbf{r}} = \mathbf{r}/r$, $J_\alpha(M_\alpha)$ is the spin (projection of the spin) of particle α, $l_\alpha(m_\alpha)$ is the relative angular orbital momentum (its projection) of particles β and p in the bound state, $\alpha = \beta + p$, $j_\alpha(\nu_\alpha)$ is the total angular momentum (its projection) of the proton in nucleus α, and $(j_1\nu_1 \; j_2\nu_2 \mid j_3\nu_3)$ is the Clebsch-Gordan coefficient. For reaction (3), $J_X = 3/2$, $J_Y = 2$, $l_Y = 1$, $j_Y = 1/2, 3/2$; $J_a = 1/2$, $J_b = 1$, $l_a = 0$, $j_a = 1/2$, $J_p = 1/2$. For $r > R_n$, where R_n is the nuclear interaction radius,

$$I_{\alpha\beta l_\alpha j_\alpha}(r) \approx C_{\alpha(l_\alpha j_\alpha)} \frac{W_{-\eta_\alpha, l_\alpha + 1/2}(2\kappa_\alpha r)}{r}. \tag{6}$$

$W_{\lambda,\delta}(\rho)$ is the Whittaker function, $\kappa_\alpha = \sqrt{2\mu_\alpha \epsilon_\alpha}$, μ_α is the reduced mass of particles β and p, ϵ_α is their binding energy in the bound state $\beta + p = \alpha$, $\eta_\alpha = z_\beta z_p \mu_\alpha / \kappa_\alpha$ is the Coulomb parameter for this bound state, Z_α is the charge of particle α, and $C_{\alpha(l_\alpha j_\alpha)}$ is the ANC of $I_{\alpha\beta l_\alpha j_\alpha}(r)$.

For simplicity in the presentation, we consider the DWBA amplitude in the zero-range (ZR) approximation for V_{bp}, although calculations were made within the framework of the finite-range DWBA. In the ZR approximation, the DWBA amplitude takes the form

$$M = \frac{1}{2\mu_{bp}} e^{i\frac{\pi}{2}\eta_a} C_{^3\mathrm{He}(0^+_{\frac{1}{2}})} \sum_{j_Y \nu_Y m_Y M_p} (\tfrac{1}{2} M_p \; 1 m_Y \mid j_Y \nu_Y)$$
$$\times (\tfrac{3}{2} M_X \; j_Y \nu_Y \mid 2 M_Y) (\tfrac{1}{2} M_p \, 1 M_b \mid \tfrac{1}{2} M_a) \int d\mathbf{r} \, \Psi_f^{*(-)}(\mathbf{r}) I_{YX1j_Y}(r) Y^*_{1m_Y}(\hat{\mathbf{r}}) \Psi_i^{(+)}(\mathbf{r}). \tag{7}$$

In the conventional ZR DWBA, the reaction amplitude is parametrized in terms of the so-called ZR normalization constant. However, it can be shown that within the framework of the three-body approach (with constituent clusters X, b, and p), the peripheral transfer reaction amplitude is parametrized in terms of the product of the ANC's for $Y \to X + p$ and $a \to b + p$ virtual decays [24]. That is why we have parametrized the amplitude M in Eq. (7) in terms of the ANC $C_{^3\mathrm{He}(0\,\frac{1}{2})}$ rather than the ZR normalization constant. If reaction (3) is surface, i.e., if the contribution from the region $r < R_n$ to the integral in Eq. (7) is negligibly small, then $I_{YX1j_Y}(r)$ in (7) can be approximated by Eq. (6) and we derive

$$M \approx \frac{1}{2\mu_{bp}} e^{i\frac{\pi}{2}\eta_a} C_{^3\mathrm{He}(0\,\frac{1}{2})} \sum_{j_Y \nu_Y m_Y M_p} C_{Y(1j_Y)}(\tfrac{1}{2}M_p\, 1m_Y \mid j_Y\nu_Y)$$

$$\times(\tfrac{3}{2}M_X\, j_Y\nu_Y \mid 2M_Y)(\tfrac{1}{2}M_p\, 1M_b \mid \tfrac{1}{2}M_a) \int_{r>R_n} d\mathbf{r}\, \Psi_f^{*(-)}(\mathbf{r}) \frac{W_{-\eta_Y,\frac{3}{2}}(2\kappa_Y r)}{r} Y^*_{1m_Y}(\hat{\mathbf{r}})\Psi_i^{(+)}(\mathbf{r}). \tag{8}$$

Then from (8) one easily gets that the differential cross section

$$\frac{d\sigma}{d\Omega} = \lambda C^2_{^3\mathrm{He}(0\,\frac{1}{2})}(C^2_{^8\mathrm{B}(1\,\frac{1}{2})} + C^2_{^8\mathrm{B}(1\,\frac{3}{2})})\tilde{\sigma}, \tag{9}$$

$$\tilde{\sigma} = \sum_{m_Y} \left| \int_{r>R_N} d\mathbf{r}\, \Psi_f^{*(-)}(\mathbf{r}) \frac{W_{-\eta_Y,\frac{3}{2}}(2\kappa_Y r)}{r} Y^*_{1m_Y}(\hat{\mathbf{r}})\Psi_i^{(+)}(\mathbf{r}) \right|^2, \tag{10}$$

where λ is the kinematical factor. It follows from Eq. (9) that for the surface reaction (3), $d\sigma/d\Omega$ is proportional to $C^2_{^3\mathrm{He}(0\,\frac{1}{2})}(C^2_{^8\mathrm{B}(1\,\frac{1}{2})} + C^2_{^8\mathrm{B}(1\,\frac{3}{2})})$. This factor can be defined by normalizing the theoretical cross section [Eq. (9)] to the experimental one. Since, as we have noted, the value of the ANC for the virtual decay $^3\mathrm{He} \to d + p$ is well established, we can extract the factor $C^2_{^8\mathrm{B}(1\,\frac{1}{2})} + C^2_{^8\mathrm{B}(1\,\frac{3}{2})}$ which calibrates the absolute normalization of the radiative capture process, $^7\mathrm{Be}+p \to {}^8\mathrm{B}+\gamma$, at astrophysical energies [12,13].

III. CALCULATIONS

We calculated, within the framework of the finite-range DWBA, the cross sections for the $^7\mathrm{Be}(^3\mathrm{He}, d)^8\mathrm{B}$ reaction at different incident energies of ^{3}He ions, E = 21, 27, 33, and 45 MeV, which correspond to 49, 63, 77, and 105 MeV, respectively, for the energies of the ^{7}Be beam that would be used for the $^3\mathrm{He}(^7\mathrm{Be}, {}^8\mathrm{B})d$ reaction. Our aim was to find an energy interval where the reaction under consideration would be essentially peripheral, so that we can neglect the contribution from the ^{8}B interior to the reaction amplitude.

When calculating the DWBA cross sections, the overlap integrals I_{YX} and I_{ab} have been approximated as

$$I_{\alpha\beta}(\mathbf{r}) = S^{1/2}_{\alpha\beta}\phi_{\beta p}(\mathbf{r}), \tag{11}$$

where $S_{\alpha\beta}$ is the spectroscopic factor for the configuration $\beta + p$ and $\phi_{\beta p}$ is the bound-state wave function of the relative motion of $\beta + p$ in nucleus α. Since we are not interested in the absolute value of the cross section, we took both spectroscopic factors equal to 1. Tombrello's potential [6] has been used to calculate the single-particle wave function $\phi_{^7\mathrm{Be}p}$ in the $1p$ proton state. We note that the exact form of the interaction potential for ^{7}Be and p in the bound state does not influence the behavior of the cross section of the peripheral reaction $^7\mathrm{Be}(^3\mathrm{He}, d)^8\mathrm{B}$ in the main stripping peak as long as it provides the correct binding energy of the (^{7}Be-p) bound state. We used the usual Woods-Saxon potential and well-depth procedure to calculate the wave function ϕ_{dp}. To investigate the peripheral character of reaction (3), we introduced the cutoff radius R_{cut} in the integral in Eq. (7). R_{cut} was varied from 0 to 7 fm to determine how the cross section for the $^7\mathrm{Be}(^3\mathrm{He}, d)^8\mathrm{B}$ reaction changed at small angles and to check whether the reaction mechanism is dominated by the nuclear surface.

The choice of the optical potentials in initial and final channels is the most serious problem since there are no data on $^3\mathrm{He}+{}^7\mathrm{Be}$ and $d + {}^8\mathrm{B}$ scattering. Thus the optical potentials adopted in this paper must be considered to be only rough estimates. For E = 33 and 45 MeV in the entrance channel, we used $^3\mathrm{He}+{}^7\mathrm{Li}$ optical potential parameters available from the literature. In fact, because of the strong cluster structure of ^{7}Be, the exchange mechanism from $^7\mathrm{Be}(^3\mathrm{He}, {}^7\mathrm{Be})^3\mathrm{He}$ transfer can contribute (perhaps significantly) to the elastic $^3\mathrm{He}+{}^7\mathrm{Be}$ scattering. Such a problem does not exist in the $^3\mathrm{He}+{}^7\mathrm{Li}$ scattering, and so the use of this optical potential in the $^7\mathrm{Be}(^3\mathrm{He}, d)^8\mathrm{B}$ calculations is perhaps more reasonable since we take into account the pure proton transfer mechanism in a proper way. Unfortunately for E = 21 and 27 MeV, there are no $^3\mathrm{He}+{}^7\mathrm{Li}$ potentials available in the literature; consequently, we used $^3\mathrm{He}+{}^9\mathrm{Be}$ optical potentials. In the final channel, $d+{}^9$Be optical potentials were

TABLE I. Ratio of the cross sections for the ^{7}Be(^{3}He, d)^{8}B reaction calculated at different energies with the cutoff radii equal to 4 fm and 0 fm. Optical parameters are taken from Table II.

E (MeV)	Optical potential	θ (deg) 1	3	9	15
		$\sigma(R_{\rm cut}=4$ fm$)/\sigma(R_{\rm cut}=0$ fm)			
21	H1-D1	1.02	1.02	1.01	0.98
27	H1-D2	0.97	1.01	1.00	1.00
	H2-D2	1.02	1.02	0.95	0.93
33	H3-D3	0.93	0.93	0.88	0.80
	H5-D3	1.02	1.03	0.97	
45	H8-D4	0.97	0.96	0.85	0.68
	H7-D4	0.87	0.86	0.79	
	H6-D5	0.74	0.73	0.66	0.60

used for all the energies. Although the ^{8}B internal structure differs strongly from ^{9}Be, both of them are weakly bound nuclei so that their optical potentials should have large diffuseness parameters.

The problem of the discrete ambiguity of the optical potentials plays an important role in these calculations. As is well known, deep potentials produce distorted waves with more oscillations in the nuclear interior than shallow potentials. These oscillations cut off the contributions from the nuclear interior. So using deep potentials should automatically suppress, to some extent, the internal contribution. That is why we are interested in finding an energy of the incident beam where the difference between the results obtained with shallow and deep potentials would be small. We show in Table I the ratio $\sigma(R_{\rm cut}=4$ fm$)/\sigma(R_{\rm cut}=0$ fm) calculated at four different energies for ^{3}He and at scattering angles (in the center-of-mass system) of $\theta = 1°, 3°, 9°$, and 15°, using the optical potentials from Table II. At $R_{\rm cut}=4$ fm, the behavior of the $I_{YX1j_Y}(r)$ differs from $W_{-\eta_Y,\frac{3}{2}}(2\kappa_Y r)/r$ only by 4%. As can be seen from Table I, the smaller the energy, the smaller the difference between cross sections calculated with the shallow and deep optical potentials. At 45 MeV for deep potentials, the contribution from the internal region ($r < 4$ fm) is only 4% at angles < 3°, while for the shallow potentials this contribution is 27% in the same angular range. But at $E = 21$–27 MeV, both deep and shallow potentials have a small contribution from the nuclear interior. At $\theta < 10°$ the maximum value of this contribution is 5%. This situation reflects the fact that at higher energies the transparency of nuclei grows and rainbow phenomena take place. Although rainbow scattering is believed to help in distinguishing between different types of ambiguities, new data in ^{3}He scattering show the presence of another type of ambiguity called the V/W ambiguity [33], which also causes some problems in the optical potential choice. The problem of choosing the optical potentials is important since it can produce up to 28% ambiguity in the cross section of the ^{7}Be(^{3}He, d)^{8}B reaction (see Table III) in the angular range $\theta < 10°$. Such an ambiguity occurs for energies where the contribution from the nuclear interior is not small, but these energies are not of interest. Unfortunately we cannot calculate the influence of the optical potential ambiguity on the cross section in the most interesting energy interval $E = 21$–27 MeV, where reaction (3) is completely peripheral, because optical potentials are not established for these energies. However, we note that the optical potential ambiguity is less important for surface reactions. Angular distributions from the calculations are shown in Fig. 1. The cross sections at all four energies are quite large and have a pronounced peak at small angles with a magnitude between 20 and 50 mb/sr.

We can estimate the accuracy of determining the ANC of the overlap integral $I_{^8{\rm B}\,^7{\rm Be}}$ [and hence the astrophysical S factor for ^{7}Be(p,γ)^{8}B reactions at solar energies] from the analysis of the ^{7}Be(^{3}He, d) ^{8}B reaction at 20–27 MeV where the discrete ambiguity is not so important as at higher energies, and Eq. (9) is valid with accuracy of 10% at $\theta < 10°$. Let us assume that the ambiguity in

TABLE II. Optical potentials for the ^{3}He+^{7}Be and $d + {}^8$B scattering. The Coulomb radius is $r_C = 1.3$ fm.

E (MeV)		V_R	r_R	a_R	W_v	r_v	a_v	W_d	r_d	a_d	Reference
						^{3}He+^{7}Be					
21,27	H1	62.7	1.432	0.746	21.8	1.525	1.055				[30]
	H2	108.1	1.08	0.857	21.7	1.636	0.893				[30]
33	H3	69.5	1.18	0.76				20.3	1.18	0.76	[30]
	H4	176.6	1.11	0.707	51.7	0.96	0.801				[31]
	H5	146.9	1.39	0.684	29.1	1.91	0.407				[31]
45	H6	79.8	1.2	0.7	8.24	2.15	1.07				[32]
	H7	66.9	1.3	0.73	21.4	1.0	1.38				[32]
	H8	129.5	1.3	0.57	7.8	1.6	1.64				[32]
						$d + {}^8$B					
12	D1[a]	78.0	0.967	1.04	30.	1.07	0.81				[33]
17	D2	65.0	1.25	0.79				7.2	1.25	1.025	[30]
22	D3	59.4	1.447	0.776	26.0	1.447	0.776				[30]
	D4	74.03	1.239	0.736				11.63	1.239	0.736	[30]
33	D5	84.7	1.11	0.609				5.29	1.37	1.003	[30]

[a]We also use the spin-orbit potential for the $D1$ parameter set with $V_{\rm s.o.} = 6.05$ MeV, $r_{\rm s.o.} = 0.967$ fm, and $a_{\rm s.o.} = 1.04$ fm [33].

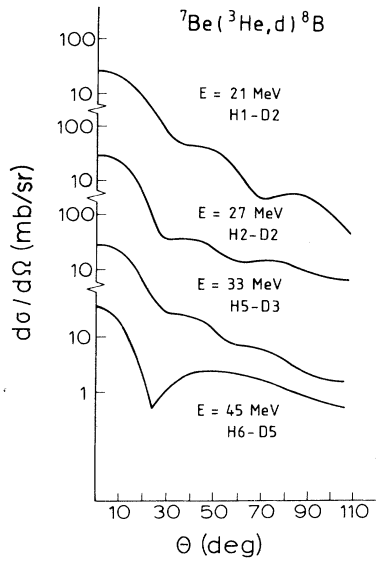

FIG. 1. Differential cross sections for the ^{7}Be(^{3}He, d)^{8}B reaction calculated within the framework of the finite-range DWBA for different ^{3}He energies, and for the optical potentials listed in Table II.

the optical potential choice gives rise to an uncertainty in the DWBA cross section of 15–20 %. If the uncertainty of the experimental cross section is 10%, then the accuracy of the $C^2_{1\frac{1}{2}} + C^2_{1\frac{3}{2}}$ and $S_{17}(0)$ is 35–40 %. To decrease this, it is necessary to improve the accuracy of the experiment. It is even more important to measure the angular distribution of the elastic scattering in the entrance and exit channels, since the main uncertainty comes from the optical potential choice. This would allow for a decrease to 25–30 % in the total uncertainty for the extracted value of $S_{17}(0)$. A more definite conclusion can be made only after analysis of the experimental data.

IV. CONCLUSION

We have shown that, in principle, the inverse reaction ^{3}He(^{7}Be, ^{8}B)d could be carried out with a radioactive ^{7}Be beam to find the ANC's described above. However, the laboratory to center-of-mass conversion for the reaction would result in a large uncertainty in scattering angle due to the finite divergence of the radioactive beam. For this reason, a better choice of reactions to study would be ones with heavier targets than ^{3}He, such as ^{10}B(^{7}Be, ^{8}B)^{9}Be or ^{14}N(^{7}Be, ^{8}B)^{13}C. Both ^{10}B and ^{14}N targets have comparatively loosely bound protons, and only one ANC, corresponding to the virtual decay ^{10}B→ ^{9}Be+p or ^{14}N→ ^{13}C+p, is needed to define the normalization of the corresponding surface reaction amplitude. We also note that by using radioactive beams, it is quite feasible that elastic scattering data can be obtained both for the entrance and exit channels, thus significantly reducing the ambiguity in the determination of the optical potentials. The ANC's for ^{10}B→ ^{9}Be+p and ^{14}N→ ^{13}C+p have been calculated [18] but not measured. Thus it is important to measure these ANC's using, for example, ^{9}Be(^{10}B, ^{9}Be)^{10}B, ^{13}C(^{14}N, ^{13}C)^{14}N, or ^{9}Be(^{3}He, d)^{10}B and ^{13}C(^{3}He, d)^{14}N reactions. Work is now underway at the TAMU Cyclotron Institute to study the ^{10}B(^{7}Be, ^{8}B)^{9}Be reaction using a radioactive ^{7}Be beam that is produced by the reaction p(^{7}Li, ^{7}Be)n and is separated from the primary beam by passing through the recoil spectrometer MARS. As part of this work, the ANC for ^{10}B→ ^{9}Be+p will be measured.

TABLE III. The dependence of the ^{7}Be(^{3}He, d)^{8}B reaction cross section on the choice of the optical potentials at E = 33 and 45 MeV.

E (MeV)	Optical potentials	θ (deg) 1	9
33	H4-D3	45.3	32.7
	H5-D3	40.4	28.8
45	H6-D4	45.3	27.6
	H8-D4	35.7	23.3
	H6-D5	44.9	27.0
	H7-D5	38.9	23.1
	H8-D5	34.2	21.6

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