Astrophysical S factor for $^9\text{Be}(p,\gamma)^{10}\text{B}$

A. Sattarov, A. M. Mukhamedzhanov, A. Azhari, C. A. Gagliardi, L. Trache, and R. E. Tribble

Cyclotron Institute, Texas A&M University, College Station, Texas 77843

The $^9\text{Be}(p,\gamma)^{10}\text{B}$ reaction plays an important role in primordial and stellar nucleosynthesis of light elements in the p shell, but the energy dependence of $S(E)$ has not been well understood. We reanalyze the existing $^9\text{Be}(p,\gamma)^{10}\text{B}$ experimental data within the framework of the R-matrix method. The direct capture part of the S factor is calculated using the experimentally measured asymptotic normalization coefficients for $^{10}\text{B}\rightarrow{}^9\text{Be}+p$. The fitted parameters of the low-lying ^{10}B resonances are also required to be consistent with previous measurements of $^6\text{Li}(\alpha,\gamma)^{10}\text{B}$. A good simultaneous fit to both radiative capture reactions is found, in contrast to previous analyses. These results demonstrate that experimentally measured asymptotic normalization coefficients, coupled to the R-matrix method, can provide a reasonable determination of direct radiative capture rates, even when the captured proton is tightly bound in the final nucleus. [S0556-2813(99)03608-0]

PACS number(s): 25.40.Lw, 24.30.Gd, 26.20.+f, 27.20.+n

I. INTRODUCTION

The reaction $^9\text{Be}(p,\gamma)^{10}\text{B}$ plays an important role in primordial and stellar nucleosynthesis of light elements in the p shell [1–5]. There are two previous measurements of the astrophysical factor $S(E)$ for this reaction at low energies [4,5] and one measurement of the analyzing power [6]. The energy dependence of this capture reaction over the range important to nuclear astrophysics is quite complex because it includes contributions from direct capture and several resonances, and they interfere with each other. Furthermore, the $^9\text{Be}(p,d)^8\text{Be}$ and $^9\text{Be}(p,\alpha)^6\text{Li}$ channels are both open at threshold, complicating theoretical efforts to construct the low-energy $^9\text{Be}+p$ optical potential required to calculate the direct capture contribution. These effects have made a detailed understanding of $S(E)$ quite difficult.

Analysis of the behavior of $S(E)$ has been performed in [4–6]. In [4], the S factor was measured over the range $68<E_{\text{c.m.}}<125$ keV by comparing yields for $^9\text{Be}(p,\alpha)^6\text{Li}$ and $^9\text{Be}(p,\gamma)^{10}\text{B}$. $S(E)$ was found to be independent of energy, although the very limited statistics admit the possibility of a significant variation of the S factor with energy. The data were analyzed assuming pure direct capture. In [5], $S(E)$ was measured over the range $66<E_{\text{c.m.}}<1620$ keV using a 4π NaI(Tl) summing crystal. The results disagree with [4] regarding both the shape and the magnitude of $S(E)$. The measurements in [5] show indications of both direct and resonant capture at low energies, and their combined presence has now been verified by the analyzing power study [6]. Meanwhile, the S factor determined in [4] is larger than the one in [5] by a factor of 4. Given these discrepancies, it is now assumed that the cross section measurements in [4] are unreliable. However, there are also inconsistencies in the more recent fits of $S(E)$ [5,6], as described below.

Recently, as part of our program to determine the $^7\text{Be}(p,\gamma)^8\text{B}$ direct capture rate at stellar energies [7], we have measured the asymptotic normalization coefficients (ANC's) for the virtual decay of the ground and low-lying excited states of ^{10}B into the channel $^9\text{Be}+p$ [8]. At stellar energies, the $^7\text{Be}(p,\gamma)^8\text{B}$ reaction is dominated by proton captures that occur well beyond the nuclear radius, and the $^7\text{Be}+p$ scattering waves are well reproduced by pure Coulomb waves as there are no inelastic channels open except the radiative capture itself. Under these conditions, we have shown [9,10] that direct radiative capture rates at low energies may be determined by measuring the corresponding ANC's in proton transfer reactions.

The simple relationship between ANC's and direct capture rates that exists for $^7\text{Be}(p,\gamma)^8\text{B}$ is not applicable to direct capture in $^9\text{Be}(p,\gamma)^{10}\text{B}$. Notably, the relatively tight binding energy of the last proton in ^{10}B ($\varepsilon=6.586$ MeV) implies that $^9\text{Be}(p,\gamma)^{10}\text{B}$ direct capture may have a significant contribution from the nuclear interior, making the simple direct capture model adopted in [9,10] break down. Nonetheless, it is interesting to investigate how well one can predict the $^9\text{Be}(p,\gamma)^{10}\text{B}$ direct capture rate from the measured ANC's, in order to test the relationship between them under adverse circumstances.

The R-matrix approach relates a direct capture rate to a radial integral which is taken from the channel radius to ∞ [11], so only the peripheral part of the channel overlap function is needed. The absolute normalization of the peripheral part of this overlap function is specified by the corresponding ANC [8]. In addition, solid-sphere scattering phase shifts are used in the R-matrix approach to take into account the nuclear scattering in the initial state. These features combine to minimize the uncertainties in the calculated $^9\text{Be}(p,\gamma)^{10}\text{B}$ direct capture rate when using this approach, making it an ideal tool for this test.

In Sec. II, we describe the calculation of $S(E)$ in the R-matrix approach. In Sec. III, we discuss the calculations of the $^9\text{Be}(p,\gamma)^{10}\text{B}$ direct capture rate, using our measured ANC's as inputs, and in Sec. IV, we describe our fit to the experimental $S(E)$. We find results that are comparable to those in [5,6]. However, unlike [5,6], our fit is also consistent with the ^{10}B resonance parameters that have been extracted from other complementary reaction studies. Finally, Sec. V contains concluding remarks.

II. *R*-MATRIX APPROACH TO RADIATIVE CAPTURE

In this section we give the explicit equation for $S(E)$ for $^9\text{Be}(p,\gamma)^{10}\text{B}$ in the *R*-matrix approach. We use the standard convention

$$S(E) = E e^{2\pi\eta}\sigma(E), \tag{1}$$

where $\sigma(E)$ is the radiative capture cross section, E is the relative kinetic energy of the $p + {}^9\text{Be}$ system, and η is the Sommerfeld parameter. We use the system of units in which $\hbar = c = 1$, and all widths are given in the center-of-mass system.

The total radiative capture cross section populating the ground and excited states of ^{10}B is given by [11]

$$\sigma(E) = \sum_{J_{n_f}J} \sigma_{J_{n_f}J}(E), \tag{2}$$

where J is the total angular momentum of the colliding particles, J_{n_f} is the spin of the n_fth bound state in ^{10}B, and

$$\sigma_{J_{n_f}J}(E) = \frac{\pi}{k^2} \frac{2J_{n_f}+1}{(2J_{^9\text{Be}}+1)(2J_p+1)} \sum_{I,l} |U_{IlJJ_{n_f}}(E)|^2. \tag{3}$$

Here, J_i is the spin of nucleus i, I is the channel spin, and l is the relative orbital angular momentum of the colliding ^9Be and p. The *R*-matrix expression for the amplitude has been derived from [12,11,13], taking into account the contributions from resonance and direct $E1$ and $M1$ captures:

$$U_{IlJJ_{n_f}}(E) = U^{(R)}_{IlJJ_{n_f}}(E) + U^{(D,E1)}_{IlJJ_{n_f}}(E) + U^{(D,M1)}_{IlJJ_{n_f}}(E). \tag{4}$$

The resonance part of the collision matrix describing proton capture into the resonance level λ with spin J and with the subsequent decay of this resonance into the bound state with quantum numbers J_{n_f} is given in the standard one-level *R*-matrix approximation by

$$U^{(R)}_{IlJJ_{n_f}}(E) = -i e^{i\xi_l} \frac{[\Gamma^p_{IlJ}(E)]^{1/2}[\Gamma^\gamma_{JJ_{n_f}}(E)]^{1/2}}{E_\lambda - E - i\Gamma_\lambda/2}. \tag{5}$$

ξ_l is the sum of the hard-sphere and Coulomb phase shifts, $\Gamma^p_{IlJ}(E)$ is the observed proton partial width of the resonance level λ with the spin J and resonance energy E_λ for decay

into the channel with the orbital angular momentum l and channel spin I, Γ_λ is the observed total width of the level λ, and $\Gamma^\gamma_{JJ_{n_f}}(E)$ is the observed γ width for the decay of the λth resonance to the n_fth bound state. The total width of the resonance is the sum of its p, d, α, and γ partial widths. The energy dependence of the proton partial width is given by

$$\Gamma^p_{IlJ}(E) = \sqrt{\frac{E}{E_\lambda}} \frac{P_l(E,r_{p0})}{P_l(E_\lambda,r_{p0})} \Gamma^p_{IlJ}(E_\lambda). \tag{6}$$

Here, $P_l(E,r_{p0})$ is the Coulomb penetration factor in the proton channel, and r_{p0} is the proton channel radius. Similar expressions give the energy dependence of the partial widths for the other particle-decay channels. The energy dependence of the gamma width, assuming dipole radiation, is given by

$$\Gamma^\gamma_{JJ_{n_f}}(E) = \left(\frac{E_\gamma}{E_\lambda + \varepsilon_{n_f}}\right)^3 \Gamma^\gamma_{JJ_{n_f}}(E_\lambda), \tag{7}$$

where $E_\lambda + \varepsilon_{n_f}$ is the energy of the emitted γ ray on resonance. The collision matrix element corresponding to the direct $E1$ capture into the n_fth bound state is given by [13,11]

$$
\begin{aligned}
U^{(D,E1)}_{IlJJ_{n_f}}(E) = {}& i^{l+1-l_{n_f}} \sqrt{\frac{2\mu}{3kr_{p0}}} k_\gamma^{3/2} e \langle l0\ 10|l_{n_f}0\rangle \\
& \times \sqrt{(2l+1)(J_{n_f}+1)}\, W(1l_{n_f}JI;lJ_{n_f}) \\
& \times N^{1/2}_{n_f} \theta_{l_{n_f}IJ_{n_f}} \frac{1}{W_{-\eta_{n_f},l_{n_f}+1/2}(2\kappa_{n_f}r_{p0})} \\
& \times \int_{r_{p0}}^\infty dr\, r W_{-\eta_{n_f},l_{n_f}+1/2}(2\kappa_{n_f}r)(I_l - e^{2i\xi_l}O_l).
\end{aligned}
\tag{8}
$$

Here $\kappa_{n_f} = \sqrt{2\mu\varepsilon_{n_f}}$, ε_{n_f} is the binding energy of the n_fth bound state of ^{10}B for the virtual decay to $^9\text{Be}+p$, μ is the reduced mass of $^9\text{Be}+p$, $I_l(r)$ and $O_l(r)$ are the incoming and outgoing solutions of the radial Schrödinger equation, $\langle j_1 m_1 j_2 m_2|jm\rangle$ is the Clebsch-Gordan coefficient, $W(j_1 j_2 j_4 j_5;j_3 j_6)$ is the standard Rakah coefficient, $W_{-\eta_{n_f},l_{n_f}+1/2}(2\kappa_{n_f}r)$ is the Whittaker function defining the behavior of the proton radial bound state wave function at $r > r_{p0}$, $\theta_{l_{n_f}IJ_{n_f}}$ is the reduced width amplitude, and N_{n_f} is the normalization factor [11,14]. We note that in the *R*-matrix method the integration of the radial matrix element starts from the channel radius r_{p0}, which is a fitting parameter. The collision matrix element corresponding to the direct $M1$ capture into the n_fth bound state is given by [13]

$$U^{(D,M1)}_{IlJJ_{n_f}}(E) = (-1)^{J+1-J_{n_f}} i^{l+1-l_{n_f}} \sqrt{\frac{2\mu}{3kr_{p0}}} k^{3/2}_\gamma \frac{e}{m} \sqrt{(2l+1)(J_{n_f}+1)}\, W(1IJl;IJ_{n_f}) \left[\sqrt{2J_p+1}\, W(1J_pIJ_{^9\text{Be}};J_pI) \sqrt{\frac{J_p+1}{J_p}}\mu_p \right.$$

$$\left. + \sqrt{2J_{^9\text{Be}}+1}\, W(1J_{^9\text{Be}}IJ_p;J_{^9\text{Be}}I) \sqrt{\frac{J_{^9\text{Be}}+1}{J_{^9\text{Be}}}}\mu_{^9\text{Be}} \right] N^{1/2}_{n_f}\theta_{l_{n_f}IJ_{n_f}} \frac{1}{W_{-\eta_{n_f},l_{n_f}+1/2}(2\kappa_{n_f}r_{p0})}$$

$$\times \int^\infty_{r_{p0}} dr\, W_{-\eta_{n_f},l_{n_f}+1/2}(2\kappa_{n_f}r)(I_l - e^{2i\xi_l}O_l), \tag{9}$$

where μ_p and μ_{Be} are the magnetic moments of the proton and ^{9}Be in nuclear magnetons.

III. DIRECT CAPTURE CALCULATIONS

In [5,6] a simple direct capture model [15,16] was used to calculate the astrophysical factor for the direct capture ^{9}Be$(p,\gamma)^{10}$B. The radial matrix element for $E1$ or $M1$ transitions was given by

$$U^{(D,K1)}_{IlJJ_{n_f}}(E) \sim (S_{l_{n_f}IJ_{n_f}})^{1/2} \int^\infty_0 dr\, r^{\mathcal{L}+2} \varphi_{l_{n_f}}(r)\psi_{lk}(r). \tag{10}$$

Here, $K=E,M$, $\mathcal{L}=1$ for $E1$ transitions, and $\mathcal{L}=0$ for $M1$ transitions. Also, $\varphi_{l_{n_f}}(r)$ is the bound state wave function of the relative motion of $p+{}^9$Be in ^{10}B calculated in the Woods-Saxon potential, $\psi_{lk}(r)$ is the optical model scattering wave function of the colliding proton and ^{9}Be, and $S_{l_{n_f}IJ_{n_f}}$ is the spectroscopic factor of the configuration ^{9}Be $+p$ with given quantum numbers in ^{10}B. In [5] only $E1$ transitions were considered, while in [6] $M1$ transitions were also included.

Equation (10) was used in [5,6] when calculating the direct part of the astrophysical factor. However, as a result of the tight binding of the last proton in ^{10}B, the contribution to ^{9}Be$(p,\gamma)^{10}$B direct capture from small r in Eq. (10) is important. When a small r contributes significantly, this simple direct capture model is not valid: many-particle effects like antisymmetrization between the incident proton and the ^{9}Be nucleons should be included and the electromagnetic transition operator should be written taking into account the interaction between the proton and individual nucleons of ^{9}Be, rather than between the proton and the center of mass of ^{9}Be. In addition, the integral is very sensitive to the parameters assumed for the optical potential. Moreover, the spectroscopic factors required for Eq. (10) have significant uncertainties [17]. In practice, [5,6] took the relative spectroscopic factors equal to those found in ^{9}Be$(d,n)^{10}$B and treated the absolute magnitudes as a fitting parameter.

In contrast, using the R-matrix method adopted here, the direct capture amplitudes contain the radial integral ranging only from the channel radius r_{p0} to infinity since the internal contribution to these amplitudes is contained within the reso-

nance part. Furthermore, the R-matrix boundary condition at the channel radius r_{p0} implies that the scattering of particles in the initial state is given by the solid-sphere phase. Hence, the problems related to the interior contribution and the choice of incident channel optical parameters do not occur. In addition, the normalization factor that appears in the direct capture amplitudes in the R-matrix approach, Eqs. (8) and (9), is related to the ANC defining the normalization of the tail of the bound state wave function of ^{10}B in the two-body channel ^{9}Be$+p$ [14,18] by

$$C_{l_{n_f}IJ_{n_f}} = \sqrt{\frac{2}{r_{p0}}}\theta_{l_{n_f}IJ_{n_f}} N^{1/2}_{n_f} \frac{1}{W_{-\eta_{n_f},l_{n_f}+1/2}(2\kappa_{n_f}r_{p0})}. \tag{11}$$

While the reduced width amplitude $\theta_{l_{n_f}IJ_{n_f}}$ depends on the channel radius, the ANC is model independent. Thus, substituting the ANC's allows one to express the direct capture amplitude in the R-matrix method in a form where the dependence on the channel radius comes only through the low integration limit, while its absolute normalization is expressed in terms of the ANC and is model independent. The channel radius in the R-matrix approach remains as a fitting parameter, but it has a clear physical interpretation. Previous works suggest that it should be comparable to or larger than the nuclear radius [12,19]. We have measured the ANC's for the ground and first three excited states of ^{10}B [8]. This makes the R-matrix approach particularly convenient to calculate the direct part contributions to the radiative capture of protons by ^{9}Be.

For the reaction under consideration, $l_{n_f}=1$ for all four bound states taken into account; for the $E1$ and $M1$ transitions, $l=0,2$ and $l=1$, respectively. We find that the contribution of the $M1$ direct capture to the S factor is negligibly small compared to the $E1$ direct capture contribution over the entire energy interval $E \leqslant 1.62$ MeV. We also find that the contribution of the $E2$ transition ($l=1$) is negligible. Thus, $E1$ completely dominates the direct capture. When calculating the $E1$ transition, only $l=0$ needs to be taken into account. The channel radius r_{p0} is the only fitting parameter. The channel radius $r_{p0}=3.1$ fm, which is slightly larger than the nuclear radius of ^{10}B, provides minimum χ^2 in fitting the data, as described in the next section. We note that the direct part turns out to be quite sensitive to the
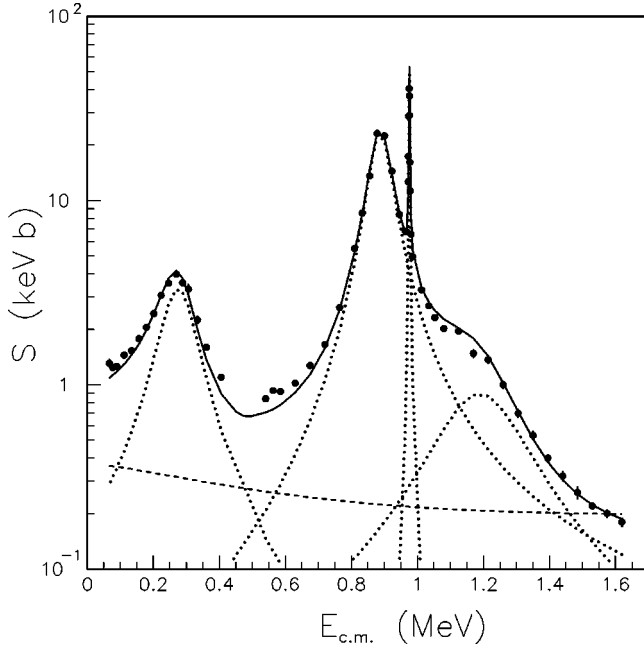
FIG. 1. The $S(E)$ factor for the reaction ^{9}Be$(p,\gamma)^{10}$B. The points are the experimental data from [5] with statistical error bars. An additional 6% uncertainty in the overall normalization must be added. The solid line is our fit assuming the second resonance $J^\pi = 2^+$. The dashed line is our calculated contribution for the direct radiative capture and the dotted lines show our calculated contribution for the resonances.

choice of the channel radius due to the high proton binding energy in ^{10}B. Changing the channel radius by 1 fm, from $r_{p0} = 3$ fm to 4 fm, decreases the total direct $S(0)$ factor by a factor of 2. Because of the behavior of the $p + \,^9$Be scattering wave function (it goes to zero at $r \rightarrow r_{p0}$) and the presence of the extra factor r from the $E1$ operator, the integrand in Eq. (8) reaches its maximum at $r = 6$ fm when the channel radius $r_{p0} = 3.1$ fm. The bound state proton wave function in ^{10}B nearly coincides with its asymptotic form $C_{l_{n_f} I J_{n_f}} W_{-\eta_{n_f}, l_{n_f} + 1/2}(2\kappa_{n_f} r)/r$ at $r \geqslant 4$ fm, so its substitution for the bound state wave function in Eq. (8) is justified. The energy dependence of the calculated astrophysical factor for the direct $E1$ capture is shown in Fig. 1, with the zero-energy direct part $S^{(D)}(0) = 0.38$ keV b. The results of the

direct capture calculations at zero energy are also given in Table I. We stress once more that, when calculating the direct capture S factors, the normalization factors for the radial matrix elements for the various final states, and for the relative $p_{3/2}$ and $p_{1/2}$ contributions for the two 1^+ states, are given by the corresponding ANC's found independently from measurements of the ^{9}Be$(^{10}$B,^{9}Be$)^{10}$B reaction [8]. Thus as noted above there is only one common fitting parameter—the channel radius r_{p0}. While carrying out fits to the data to minimize χ^2, ANC's were allowed to vary within their range of uncertainty as found in [8] and this is reflected in the uncertainties quoted in Table I. In addition, resonance parameters were allowed to vary as discussed below. We find that the total direct capture $S(0)$ factor in our fit coincides with the two previous fits [5,6] in spite of the very different procedure adopted here.

IV. FITTING $S(E)$

According to Eqs. (3) and (4), the total S factor is given by the sum of the resonance, direct capture, and interference terms. The analysis of the experimental data has been done in [5,6], taking into account the contributions from captures to three broad resonances at $E_1 = 287$ keV $(J^\pi = 1^-)$, $E_2 = 892$ keV $(J^\pi = 2^{(\pm)})$, and $E_4 = 1161$ keV $(J^\pi = 2^-)$, and one narrow resonance at $E_3 = 975$ keV $(J^\pi = 0^+)$, in addition to the direct capture and interference contributions. When analyzing the data the main problem is to get a fit at energies near the first resonance. The experimental value of $S = 3.98 \pm 0.12$ keV b at $E = 269$ keV, corresponding to the first resonance, is lower than the S factor calculated with the standard resonance parameters from compilation [17] by a factor of 4 (see Table II).

To interpret the experimental data, and especially to explain the low values of the S factor near the first resonance, the parameters of the first ^{9}Be$+p$ resonance were changed significantly in [5] compared to those determined previously from ^{9}Be$(p,\gamma)^{10}$B, ^{9}Be$(p,p)^9$Be, and ^{6}Li$(\alpha,\gamma)^{10}$B data [17]. Meanwhile, in [6] resonance parameters were adopted that were closer to the previously determined values, but the rapid energy dependence of the proton partial width in the vicinity of the first resonance was neglected.

The ^{6}Li$(\alpha,\gamma)^{10}$B radiative capture reaction is particularly important when analyzing the first ^{9}Be$+p$ resonance in ^{10}B,

TABLE I. The calculated astrophysical S factors (keV b) for the radiative capture ^{9}Be$(p,\gamma)^{10}$B at $E = 0$ keV. DC and RC stand for direct capture and resonance capture. The results for the total $S(0)$ factor presented in columns 6 and 7 assume the second resonance $J^\pi = 2^+$ and 2^-, respectively.

^{10}B final state E_x [MeV]	Zahnow *et al.* [5]	Wulf *et al.* [6] DC	DC + RC	Present analysis DC	DC + RC for 2^+	DC + RC for 2^-
0.0		0.19(1)	0.25(1)	0.21(2)	0.35(4)	0.42(4)
0.72		0.14(1)	0.34(1)	0.11(1)	0.25(3)	0.22(2)
1.78		0.03(1)	0.27(1)	0.030(3)	0.26(3)	0.26(3)
2.15		0.02(1)	0.10(1)	0.030(3)	0.10(1)	0.09(1)
Total	1.0(1)	0.38(2)	0.96(2)	0.38(2)	0.96(6)	1.00(6)

TABLE II. The fit parameters (in the center-of-mass system) for the S factor of the ^{9}Be$(p,\gamma)^{10}$B reaction. The results given in columns 5 and 6 assume the second resonance $J^\pi=2^+$ and 2^-, respectively

Resonance parameters	Compilation [17]	Zahnow et al. [5]	Wulf et al. [6]	Present results	
				for 2^+	for 2^-
J^π	1^-	1^-	1^-	1^-	1^-
E_1 [keV]	287 ± 5	342 ± 27	295	296	296
Γ_1 [keV]	120 ± 5 and 140	297 ± 27	145	140	140
Γ^γ [eV]	4.8	4.8	1.8	1.2 [a]	1.2 [a]
Γ^p/Γ_1	0.3		0.3	0.35	0.35
Γ^α/Γ^p	1.25 ± 0.12			1.35	1.35
J^π	$2^{(\pm)}$	2^-	2^+	2^+	2^-
E_2 [keV]	892 ± 2	890 ± 1.8	890	890	891
Γ_2 [keV]	74 ± 4	81.0 ± 2.7	79.2	80	80
Γ^p/Γ_2	0.90 ± 0.05			0.75	0.72
Γ^γ [eV]	25.8			25.8 [b]	25.8 [b]
J^π	0^+	0^+	0^+	0^+	0^+
E_3 [keV]	972	972	972	972 [b]	972 [b]
Γ^p/Γ_3	1.0	1.0	1.0	1.0 [b]	1.0 [b]
Γ^γ [eV]	8.5	8.5	8.5	8.5 [b]	8.5 [b]
J^π	$(1,2)^-$	2^-	2^-	2^-	2^-
E_4 [keV]	$(1206\pm18, 1161)$	1265	1215	1196	1158
Γ_4 [keV]	$(260\pm30, 210\pm60)$	387 ± 27	190	290	229
Γ^p/Γ_4	$(0.90\pm0.05,0.65)$		0.72	0.52	0.38
Γ^γ [eV]	8.5		5.8	7.9 [c]	7.9 [c]
ϕ (deg) [d]		44 ± 15			132

[a]This parameter was taken from [20] and fixed during fitting.
[b]This parameter was taken from compilation [17] and fixed during fitting.
[c]This is the partial γ width for transitions to the first four bound states of ^{10}B [17]. It was fixed during fitting.
[d]The phase factor ϕ takes into account the influence of distant resonances.

but neither of the previous fits [5,6] considered it. The ^{6}Li $+\alpha$ threshold is lower than the ^{9}Be$+p$ threshold by 2.13 MeV, so the first ^{9}Be$+p$ resonance is narrow in the α-^{6}Li channel—the ratio of the total width to the resonance energy in this channel is 0.05. This substantially reduces the ambiguities in the resonance parameters that exist in the p-^{9}Be channel because of the rapidly changing penetrability factor. The ^{6}Li$(\alpha,\gamma)^{10}$B cross section is 1.80 ± 0.4 μb at the resonance peak, and the total width of the resonance is $\Gamma_1 = 120\pm5$ keV [20]. In contrast, the parameters for the first resonance adopted in [5] would imply a total width of 230–240 keV, while those adopted in [6] would require a peak resonance cross section of 2.65 μb.

We fit the experimental data for the ^{9}Be$(p,\gamma)^{10}$B S factor and the excitation function of ^{6}Li$(\alpha,\gamma)^{10}$B [20] near the first resonance simultaneously. We include all four low-lying ^{10}B resonances in the energy region $66\le E \le 1620$ keV. The contribution of each resonance is calculated using Eq. (5) for the resonance collision matrix. The energy dependences of all the widths are taken into account. The negative parity resonances can decay into all four low-lying ^{10}B bound states. The 0^+ resonance is of the Breit-Wigner type and decays into the first and third excited

states of ^{10}B. The relative intensities of the decays of each resonance to the ground and three first excited states are taken from Table 10.11 in [17]. The absolute value of the gamma width for each resonance, except for the first, is also taken from [17] and is given in Table II. For the first resonance, we adopt $\Gamma^\gamma=1.2$ eV which, while inconsistent with the adopted value in [17], is consistent with [20] and with one of the solutions reported in [21].

Since $J_{^9\text{Be}}=3/2^-$ and $J_p=1/2^+$, the negative parity resonances are formed predominantly by s-wave capture, i.e., $l=0$ in Eq. (5). As we have indicated, the direct terms are formed predominantly through $E1$ capture ($M1$ and $E2$ direct captures can be neglected), i.e., $l=0$ in Eq. (8). Then it is clear from Eq. (3) that the direct and resonance terms with the same channel spin will interfere.

There are two possible spin-parity assignments for the second resonance, 2^- and 2^+ [17]. We performed fits for both parities. When fitting with 2^-, we introduced an additional energy-independent phase ϕ into the interference term between the second and the fourth resonances. This phase takes into account the influence of distant resonances [5]. We find that the fit with $J^\pi=2^-$ ($\chi_\nu^2=7.8$) is slightly better than that with $J^\pi=2^+$ ($\chi_\nu^2=13.2$). Thus it is difficult to select

TABLE III. The branching ratios for the radiative capture ^{9}Be$(p,\gamma)^{10}$B at $E = 83$ keV, populating the different bound states of ^{10}B. The results given in columns 5 and 6 assume the second resonance $J^\pi = 2^+$ and 2^-, respectively.

^{10}B final state E_x [MeV]	Cecil *et al.* [4]	Wulf *et al.* [6]		Present results	
		Experiment	Fit	for 2^+	for 2^-
0.0	0.22	0.24(1)	0.21	0.30	0.36
0.72	0.33	0.31(1)	0.34	0.26	0.23
1.78	0.33	0.33(1)	0.33	0.33	0.32
2.15	0.11	0.13(1)	0.12	0.11	0.10

the parity of the second resonance only from the fit of the S factor for ^{9}Be$(p,\gamma)^{10}$B. However, the fit with 2^+ gives better agreement with the measured branching ratios of the transitions to the different ^{10}B bound states (see Table III), and measurements of the low-energy ^{9}Be$(p,\gamma)^{10}$B analyzing powers strongly favor 2^+ for the second resonance [6]. For these reasons, we also adopt $J^\pi = 2^+$ for the second resonance, but we present the results of the fit for both parities in the tables.

The result of the fit is shown in Fig. 1 and the parameters of the fit are given in Table II. The best fit is derived for channel radius $r_{p0} = 3.1$ fm. Even though near the resonance peaks the resonance terms dominate, the weight of the direct part turns out to be very important, especially as $E \to 0$. In Table III the calculated relative γ-ray branching ratios at $E = 83$ keV are compared to previous measurements [4,6] and calculations [6]. Our fit slightly overestimates the relative transition rate to the ground state and underestimates the relative transition rate to the first excited state. It agrees with experiment quite well for the transitions to the second and third excited states. We note once more that the relative contributions of the direct transitions to the different bound states in our calculations are entirely determined by the ANC's extracted from the independent measurements of the ^{9}Be$(^{10}$B,^{9}Be$)^{10}$B reaction.

Using the fitted parameters from ^{9}Be$(p,\gamma)^{10}$B, we can calculate the cross section for ^{6}Li$(\alpha,\gamma)^{10}$B in the peak corresponding to the first resonance in ^{9}Be$+p$. We find $\sigma = 1.76$ μb, in good agreement with experiment. The width of this resonance is $\Gamma_1 = 140$ keV, which coincides with the apparent width extracted from the (p,γ) excitation function and is close to that inferred from the (α,γ) excitation function. Thus, our fit gives reasonable agreement with the data from both radiative capture reactions simultaneously.

V. CONCLUSION

We have reanalyzed the measured astrophysical factor $S(E)$ for the radiative capture ^{9}Be$(p,\gamma)^{10}$B within the framework of the R-matrix method, taking into account the resonance parameters found previously from measurements of the ^{9}Be$(p,\gamma)^{10}$B, ^{9}Be$(p,p)^9$Be, ^{9}Be$(p,d)^8$Be, ^{9}Be$(p,\alpha)^6$Li, and ^{6}Li$(\alpha,\gamma)^{10}$B reactions [17]. Special attention has been given to the ^{6}Li$(\alpha,\gamma)^{10}$B reaction when fitting the behavior of the S factor near the first ^{9}Be$+p$ resonance. The relative contribution of the direct transitions to the first four bound states of ^{10}B are fixed by parametrizing the direct capture amplitudes in the R-matrix approach in terms of the previously measured ANC's [8]. The only fitting parameter when calculating the direct capture contribution to the S factor in the R-matrix approach we use is the channel radius r_{p0}. The channel radius that provides the best fit of the data is slightly larger than the nuclear radius of ^{10}B, which is quite reasonable in the R-matrix approach. We find, as in two previous fits [5,6], that the direct part is important at zero energy. Despite using different methods, all three fits find consistent results for the direct contribution to the total S factor at zero energy. However, unlike the previous fits, our fit also provides a good simultaneous match to the observed ^{9}Be$(p,\gamma)^{10}$B and ^{6}Li$(\alpha,\gamma)^{10}$B cross sections in the vicinity of the first resonance in ^{9}Be$+p$. These results demonstrate that the ANC approach, coupled to the R-matrix method, can provide a reasonable determination of direct radiative capture rates, even when the captured proton is tightly bound in the final nuclear state.

ACKNOWLEDGMENTS

[1] W.A. Fowler, Rev. Mod. Phys. **56**, 149 (1984).

[2] C. Rolfs and W.S. Rodney, *Cauldrons in the Cosmos* (University of Chicago Press, Chicago, 1988).

[3] G.R. Caughlan and W.A. Fowler, At. Data Nucl. Data Tables **40**, 283 (1988).

[4] F.E. Cecil, D. Ferg, H. Liu, J.C. Scorby, J.A. McNeil, and P.D. Kunz, Nucl. Phys. **A539**, 75 (1992).

[5] D. Zahnow, C. Angulo, M. Junker, C. Rolfs, S. Schmidt, W.H. Schulte, and E. Somorjai, Nucl. Phys. **A589**, 95 (1995).

[6] E.A. Wulf, M.A. Godwin, J.F. Guillemette, C.M. Laymon, R.M. Prior, B.J. Rice, M. Spraker, D.R. Tilley, and H.R. Weller, Phys. Rev. C **58**, 517 (1998).

[7] A. Azhari, V. Burjan, F. Carstoiu, H. Dejbakhsh, C.A. Gagliardi, V. Kroha, A.M. Mukhamedzhanov, L. Trache, and

R.E. Tribble, Phys. Rev. Lett. **82**, 3960 (1999).

[8] A.M. Mukhamedzhanov, H.L. Clark, C.A. Gagliardi, Y.-W. Lui, L. Trache, R.E. Tribble, H.M. Xu, X.G. Zhou, V. Burjan, J. Cejpek, V. Kroha, and F. Carstoiu, Phys. Rev. C **56**, 1302 (1997).

[9] H.M. Xu, C.A. Gagliardi, R.E. Tribble, A.M. Mukhamedzhanov, and N.K. Timofeyuk, Phys. Rev. Lett. **73**, 2027 (1994).

[10] C.A. Gagliardi, R.E. Tribble, A. Azhari, H.L. Clark, Y.-W. Lui, A.M. Mukhamedzhanov, A. Sattarov, L. Trache, V. Burjan, J. Cejpek, V. Kroha, S. Piskor, and J. Vincour, Phys. Rev. C **59**, 1149 (1999).

[11] F.C. Barker and T. Kajino, Aust. J. Phys. **44**, 369 (1991).

[12] A.M. Lane and R.G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).

[13] R.J. Holt, H.E. Jackson, R.M. Laszewski, J.E. Monahan, and J.R. Specht, Phys. Rev. C **18**, 1962 (1978).

[14] F.C. Barker, Nucl. Phys. **A588**, 693 (1995).

[15] R.F. Christy and I. Duck, Nucl. Phys. **A24**, 89 (1961).

[16] C. Rolfs, Nucl. Phys. **A217**, 29 (1973).

[17] F. Ajzenberg-Selove, Nucl. Phys. **A490**, 1 (1988).

[18] A.M. Mukhamedzhanov and R.E. Tribble, Phys. Rev. C **59**, 3418 (1999).

[19] J.M. Blatt and V.F. Weisskopf, *Theoretical Nuclear Physics* (Springer-Verlag, New York, 1979).

[20] W. Auwärter and V. Meyer, Nucl. Phys. **A242**, 129 (1975).

[21] A.B. Clegg, Philos. Mag. **1**, 1116 (1956).