

Astrophysical S factor for $^{13}\text{C}(p, \gamma)^{14}\text{N}$ and asymptotic normalization coefficientsA. M. Mukhamedzhanov,¹ A. Azhari,¹ V. Burjan,² C. A. Gagliardi,¹ V. Kroha,² A. Sattarov,¹ X. Tang,¹
L. Trache,¹ and R. E. Tribble¹¹*Cyclotron Institute, Texas A&M University, College Station, Texas 77843*²*Institute for Nuclear Physics, Czech Academy of Sciences, Prague-Rez, Czech Republic*

(Received 13 February 2002; published 2 August 2002)

We reanalyze the $^{13}\text{C}(p, \gamma)^{14}\text{N}$ radiative capture reaction within the R -matrix approach. The low-energy astrophysical S factor has important contributions from both resonant and nonresonant captures. The normalization of the nonresonant component of the transition to a particular ^{14}N bound state is expressed in terms of the asymptotic normalization coefficient (ANC). In the analysis we use the experimental ANC's inferred from the $^{13}\text{C}(^{14}\text{N}, ^{13}\text{C})^{14}\text{N}$ and $^{13}\text{C}(^3\text{He}, d)^{14}\text{N}$ reactions. The fits of the calculated S factors to the experimental data are sensitive to the ANC values and are used to test the extracted ANC's. We find that for transitions to all the states in ^{14}N , except the third excited state, the ANC's determined from the transfer reactions provide the best fit. The astrophysical factor we obtain, $S(0) = 7.7 \pm 1.1$ keV b, is in excellent agreement with previous results.

DOI: 10.1103/PhysRevC.66.027602

PACS number(s): 25.40.Lw, 24.30.Gd, 26.20.+f, 27.20.+n

I. INTRODUCTION

The $^{13}\text{C}(p, \gamma)^{14}\text{N}$ reaction is one of the important reactions in the CNO cycle. It is a predecessor of the slowest reaction in the CNO cycle, $^{14}\text{N}(p, \gamma)^{15}\text{O}$, which defines the rate of energy production in the cycle [1]. In addition, the $^{13}\text{C}(p, \gamma)^{14}\text{N}$ radiative capture rate is important for nucleosynthesis via the s process because it can deplete the seed nuclei required for the neutron generator reaction $^{13}\text{C}(\alpha, n)^{16}\text{O}$ [2,3]. The most accurate and thorough measurements of the S factor, including captures to the first six excited states of ^{14}N , were carried out by King *et al.* [4] in the energy range $E = 100\text{--}900$ keV. Their extrapolation of the total S factor down to stellar energies gave $S(0) = 7.64$ keV b and $S(25 \text{ keV}) = 7.7 \pm 1.0$ keV b. This was higher than the recommended value because the contribution from excited states was found to be larger than previously expected [5].

There are two low lying resonances in ^{14}N at center-of-mass energies $E_{R1} = 417.9$ keV ($J^\pi = 2^-, T=0$) and $E_{R2} = 518.9$ keV ($J^\pi = 1^-, T=1$) [6,4]. The first resonance is very narrow and does not affect the astrophysically important region. The second resonance dominates the capture cross section in the energy region around and below E_{R2} down to zero energy. There is also a very wide resonance in ^{14}N at $E_{R3} = 1232$ keV ($J^\pi = 0^-, T=1$), but its effect on the low-energy S factor is small. The $^{13}\text{C}(p, \gamma)^{14}\text{N}$ reaction populates the first seven bound states of ^{14}N , proceeding through resonant and nonresonant captures. In the conventional analysis applied in Ref. [4], the nonresonant capture amplitudes were parametrized in terms of spectroscopic factors that were treated as free parameters. In addition, Ref. [4] used the traditional potential model. However, when it is applied to captures to tightly bound states such as those in ^{14}N , the capture amplitudes are sensitive to the assumed initial state optical potentials. Also antisymmetrization effects, which are typically neglected, are important due to the contribution from small radii. The dependence of the interference between the

nonresonant and resonant amplitudes on channel spin was also neglected in the analysis done in Ref. [4].

Here, we reanalyze the experimental data of Ref. [4] within the framework of the R -matrix approach. The R -matrix nonresonant amplitudes are parametrized in terms of the asymptotic normalization coefficients (ANC's) for $^{13}\text{C} + p \leftrightarrow ^{14}\text{N}$. We show that this reduces the uncertainty in the nonresonant capture amplitudes to their dependence on the channel radius. It also provides a means to test the experimental values of the ANC's, which were derived in previous proton transfer reaction studies [7–9]. Our analytical form of the astrophysical S factor reproduces the experimental data in the energy interval 105–876 keV (in the c.m.) for the transitions to each bound state. We extrapolate this expression down to zero energy to determine the $S(0)$ and $S(25)$ factors. In the analysis we neglected the contribution of the first resonance because it is so narrow that it does not affect the low-energy S factor. The nonresonant transitions to the ground and all the first six excited bound states are dominated by $E1$ transitions [4]. The second resonance can decay into the ground and first five excited states in ^{14}N , thus contributing to the corresponding resonant captures. In our fit, as in Ref. [4], we assumed that the third resonance contributed only to radiative capture to the ground state. The parameters of the second and third resonances were allowed to vary in the intervals reported in Refs. [4,6].

A crucial fitting parameter in the R -matrix method is the channel radius a . We find that $a = 5.0$ fm provides the best fit for all the transitions to the ground and first five excited states. For the transitions to the sixth excited state, we find a better fit with $a = 4.0$ fm. The interference between the resonant and nonresonant amplitudes occurs only for the transitions to the ground, first, and second excited states. For the transitions to excited states beyond the second excited state, the S factor is given by the incoherent sum of the resonant and nonresonant captures, and hence is expressed in terms of the observable resonance widths. Other fitting parameters are the resonance energies, widths, radiative widths, and ANC's. The resonance energies, widths, and radiative widths for transitions to each bound state were varied within the inter-

TABLE I. The ANC's for $^{13}\text{C}+p \rightarrow ^{14}\text{N}$. The J_f^π, E_f are given in the first column for states in ^{14}N ; the corresponding proton orbital and total angular momenta, l_f and j_f , are given in the second column. The ANC's determined from the $^{13}\text{C}(^{14}\text{N}, ^{13}\text{C})^{14}\text{N}$ and $^{13}\text{C}(^3\text{He}, d)^{14}\text{N}$ reactions [7–9] are given in the third column. The superscripts a and b denote the ANC's determined from the ^{14}N and ^3He induced reactions, respectively, while the superscript $a+b$ indicates those that come from the average of the two experiments. The ANC's providing the best fit to the S factors are presented in the last column.

State ^{14}N J_f^π, E_f (MeV)	Proton orbitals l_{fj_f}	ANC's [7–9] C^2 (fm^{-1})	Present C^2 (fm^{-1})
$1^+, 0.00$	$p_{1/2}$	$18.2 \pm 0.9^{a+b}$	18.2
	$p_{3/2}$	$0.91 \pm 0.14^{a+b}$	0.91
$0^+, 2.31$	$p_{1/2}$	8.9 ± 0.9^a	8.9
$1^+, 3.948$	$p_{1/2}$	$2.88 \pm 0.17^{a+b}$	2.71
$0^-, 4.915$	$s_{1/2}$	12.66 ± 0.89^b	33.0 ± 3.81
$2^-, 5.106$	$d_{5/2}$	0.40 ± 0.03^a	0.43
$1^-, 5.690$	$s_{1/2}$	10.33 ± 0.72^b	10.95
$3^-, 5.834$	$d_{5/2}$	0.19 ± 0.02^a	0.21

vals determined in Ref. [4]. We find that the best fit occurs when all the fitting parameters are taken within the corresponding intervals found in Ref. [4]. In principle, the ANC's can be inferred directly from the fit to the experimental S factors for each transition. However, the uncertainties of the ANC's determined from such fits are quite large due to the experimental uncertainties and resonance contributions. In this paper we use another approach. As noted above, the ANC for the synthesis $B+b \rightarrow A_n$ leading to the n th bound state of nucleus A defines the absolute normalization of the nonresonant capture S factor to this state. The ANC's for the captures to the ground and first six excited states in ^{14}N have been measured in the $^{13}\text{C}(^{14}\text{N}, ^{13}\text{C})^{14}\text{N}$ [7] and $^{13}\text{C}(^3\text{He}, d)^{14}\text{N}$ [8,9] proton transfer reactions. Comparison to the capture data allows us to check these ANC's. The ANC's for each state were varied within the uncertainty intervals found from the transfer reactions as shown in Table I. For all the states, except the third excited state at $E_f = 4.92$ MeV as discussed below, the best fit was provided by the ANC's determined from the nonresonant proton transfer reactions. The final column in Table I gives the ANC's determined by the fits to the appropriate S factors. Since the uncertainty intervals for the ANC's found from the transfer reactions are significantly smaller than the uncertainties of the data [4], the uncertainties of the ANC's providing the best fit for all the states, except for the third excited state, are given by the uncertainties determined from the transfer reactions. To estimate the uncertainty of the fit, the ANC's were varied as noted above and the resonance parameters were varied within the intervals determined in Ref. [4]. The channel radius was varied by 20% from the central value $r_0 = 5$ fm that provides the best fit. We took into account also the 11.3% systematic uncertainty of the experimental data reported in Ref. [4]. The results of the fit of the S factors are displayed in Figs. 1 and 2.

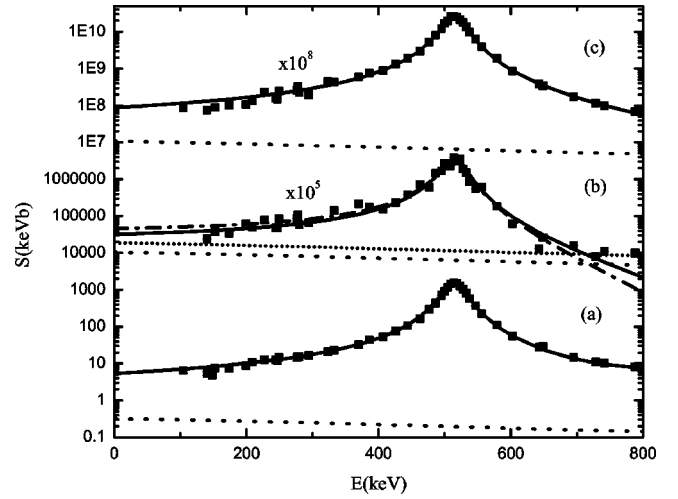


FIG. 1. $^{13}\text{C}(p, \gamma)^{14}\text{N}$ astrophysical S factors for the capture to (a) the ground state ($1^+, 0.00$ MeV), (b) the first excited state ($0^+, 2.31$ MeV), and (c) the second excited state ($1^+, 3.948$ MeV). The squares are the data points [4]. In (a) and (c), the solid line gives the calculated S factor, which includes the resonant and nonresonant capture terms and their interference, and the dotted line is the nonresonant capture S factor only. In (b), the solid line and dashed-dotted lines are the fits calculated using the ANC's $C^2 = 8.37 \text{ fm}^{-1}$ [7,8] and $C^2 = 16.0 \text{ fm}^{-1}$ [8], respectively. The two dotted lines are the nonresonant capture S factors calculated with the ANC's $C^2 = 8.37 \text{ fm}^{-1}$ (wide spacing) [7,8] and $C^2 = 16.0 \text{ fm}^{-1}$ [8] (narrow spacing).

The best fit to the ground state gives $S(0) = 5.36 \pm 0.71$ keVb. Although the nonresonant S factor gives a small contribution at zero energy, $S^{(NR)}(0) = 0.32$ keVb, its contribution is important due to interference with the resonant amplitude. The calculated resonant S factor at zero energy is $S^{(R)}(0) = 3.72$ keVb and the interference contribution is $S^{(Int)}(0) = 1.32$ keVb or 25% of the total. The uncertainty of the fit takes into account the uncertainties of the resonance parameters determined in Ref. [4], ANC's, the channel radius and experimental systematic and statistical uncertainties. The inclusion of an s -wave background pole, which interferes with the direct and second resonance terms, improves the fit slightly at energies above the second resonance peak, but makes the fit somewhat worse at energies below 400 keV. An additional uncertainty of 9% has been included in the extrapolation to zero energy of the total S factor due to the background pole. The results of the fit of the S factor are displayed in Fig. 1(a).

Capture through the second resonance to six excited states in ^{14}N contributes to the total S factor. With the exception of the first excited state, which has isospin $T = 1$, all transitions are to $T = 0$ final states. The nonresonant capture to the first excited state is an $E1$ transition. The ANC extracted for this state from the $^{13}\text{C}(^{14}\text{N}, ^{13}\text{C})^{14}\text{N}$ [7] and $^{13}\text{C}(^3\text{He}, d)^{14}\text{N}$ [8] proton transfer reactions differ by nearly a factor of 2. We attempted fitting the experimental data with both values of the ANC. The results are presented in Fig. 1(b). The ANC, $C^2 = 8.9 \pm 0.9 \text{ fm}^{-1}$, determined from the heavy ion reaction [7] provides a better fit than $C^2 = 16.0 \pm 1.1 \text{ fm}^{-1}$ deter-

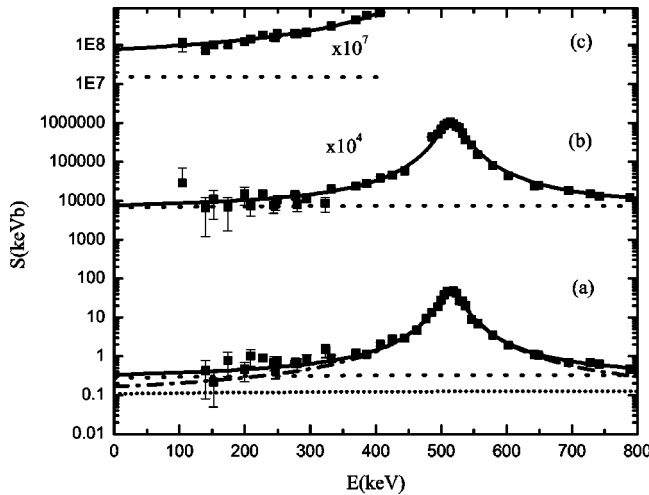


FIG. 2. $^{13}\text{C}(p, \gamma)^{14}\text{N}$ astrophysical S factors for capture (a) to the third excited state ($0^-, 4.915$ MeV) and (b) to the fifth excited state ($1^-, 5.691$ MeV). The squares are the data points [4]. In (a), the solid line and dashed-dotted lines are the fits calculated using the ANC's $C^2 = 25.4 \text{ fm}^{-1}$ and $C^2 = 12.66 \text{ fm}^{-1}$ [9], respectively. The two dotted lines are the nonresonant capture S factors calculated with the ANC's $C^2 = 25.4 \text{ fm}^{-1}$ (wide spacing) and $C^2 = 12.66 \text{ fm}^{-1}$ (narrow spacing) [9]. In (b), the solid and dotted lines are the calculated total and nonresonant S factors, respectively. The total low-energy astrophysical S factors for the $^{13}\text{C}(p, \gamma)^{14}\text{N}$ capture to the ground and first six excited states in ^{14}N are shown in (c). The solid and dotted lines are the total and total nonresonant S factors, respectively.

mined from the ($^3\text{He}, d$) reaction [8], especially at energies above the resonance. We conclude that the lower value is more appropriate and obtain for the transition to the first excited state $S(0) = 0.32 \pm 0.08$ keV b, with the nonresonant term contributing about 33% to the total S factor.

The nonresonant capture to the second excited state proceeds through an $E1$ transition. The result of the S factor fit for the transition to the second excited state is displayed in Fig. 1(c). The fit for the transition to the second excited state gives total $S(0) = 0.88 \pm 0.12$ keV b, with $S^{(NR)}(0) = 0.11$ keV b, $S^R(0) = 0.37$ keV b, and $S^{(Int)}(0) = 0.40$ keV b. The sharp drop in S at higher energies justifies the assumption that the radiative width for the decay of the third resonance to the second excited state is so small that its contribution can be neglected. The inclusion of the s -wave background pole, which interferes with the direct term and provides the best fit, only slightly improves the overall fit but it degrades the fit at higher energies. The uncertainty due to the background pole is 9% and is taken into account in the total S factor uncertainty.

The ground state and first two excited states are of positive parity and both resonant and nonresonant capture to them are dominated by $E1$ transitions. The remaining four excited states are all of negative parity. The nonresonant capture transitions to these states proceed through $E1$ transitions, but the resonant captures proceed through either $M1$ or $E2$ transitions. Generally the fits are very similar assuming

either $M1$ or $E2$ transitions, making it difficult to distinguish between the two. The results of the fit to the third excited state are shown in Fig. 2(a). This state was weakly populated in the $^{13}\text{C}(^{14}\text{N}, ^{13}\text{C})^{14}\text{N}$ reaction due to an angular momentum mismatch and could not be resolved from the fourth excited state [7]. However, it was resolved in the $^{13}\text{C}(^3\text{He}, d)^{14}\text{N}$ reaction and an ANC, $C^2 = 12.66 \pm 0.89 \text{ fm}^{-1}$ [9], was extracted assuming a $2s_{1/2}$ configuration for the transferred proton in ^{14}N . The best fit to the capture data and the fit using the ANC from Ref. [9] are both shown in Fig. 2(a). The best fit is achieved with $C^2 = 33.0 \text{ fm}^{-1}$. The uncertainty of the ANC determined from the data is $\sim 15\%$. With this value, the total S factor for the transition to the third excited state at zero energy is $S(0) = 0.33 \pm 0.07$ keV b, with $S^{(NR)}(0) = 0.28$ keV b and $S^R(0) = 0.06$ keV b.

We obtain an excellent fit to the data for the transition to the fourth excited state including resonant and nonresonant captures, with a magnitude given by the ANC from Ref. [7], and taking into account a background pole, that interferes with the second resonance destructively at energies $E < 0.517$ MeV. The calculated total S factor for this transition is $S(0) = 0.045 \pm 0.009$ keV b, where $S^{(NR)}(0) = 0.027$ keV b and $S^R(0) = 0.018$ keV b.

As for the third excited state, the ANC for the fifth excited state was determined only from the data obtained in the $^{13}\text{C}(^3\text{He}, d)^{14}\text{N}$ reaction giving $C^2 = 10.33 \pm 0.72 \text{ fm}^{-1}$ [9]. With this result, the direct and resonant capture contributions to the fifth excited state were calculated, and the results are shown in Fig. 2(b). In contrast to the third excited state, the ANC obtained from the transfer reaction gives a good accounting of the direct capture for this transition. The calculated total S factor for the transition to the fifth excited state is $S(0) = 0.77 \pm 0.09$ keV b, with $S^{(NR)}(0) = 0.66$ keV b and $S^R(0) = 0.11$ keV b.

The sixth excited state is the highest energy level populated in the $^{13}\text{C}(p, \gamma)^{14}\text{N}$ radiative capture process. Very little is known about this transition. The calculated nonresonant capture S factor for this transition is $S(0) = 0.031 \pm 0.007$ keV b which contributes less than 0.5% to the total $S(0)$ factor.

The total S factor is the sum of the individual S factors for the transitions to the ground and the first six excited states. Our result for the low-energy part of the total S factor is shown in Fig. 2(c). The value obtained for the total S factor at zero energy is $S(0) = 7.7 \pm 1.1$ keV b. This is in excellent agreement with the result found in the analysis by King *et al.* [4]. However, it is important to note that our result is obtained by extrapolating the appropriate analytic form of the S factor for each final state to zero energy. In contrast, King *et al.* used an empirical parametrization of the energy dependence of the S factor to obtain their extrapolation to zero energy. We find that the individual contributions to the total S factor are $S^{(NR)}(0) = 1.54$ keV b, $S^R(0) = 4.33$ keV b, and $S^{(INT)}(0) = 1.87$ keV b. At $E = 25$ keV, the total calculated S factor is $S(25) = 8.2 \pm 1.2$ keV b.

The reaction rates for $^{13}\text{C}(p, \gamma)^{14}\text{N}$ calculated using our

$S(E)$ factor for temperatures $T_9 < 1.0$ defining the hydrostatic hydrogen burning regime are, on average, 1.17 times higher than the reaction rates given in the latest compilation [10]. Thus we confirm the results indicating higher rates of ^{13}C consumption. This higher rate reduces the amount of ^{13}C available for the $^{13}\text{C}(\alpha, n)^{16}\text{O}$ reaction, thus making it less effective as a neutron source to fuel heavy element synthesis through the s process.

ACKNOWLEDGMENTS

We acknowledge Professor J. D. King for communicating the experimental data to us. This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG03-93ER40773, the U.S. National Science Foundation under Grant No. INT-9909787, ME 385(2000) project NSF and MSMT, CR, Grant No. GACR 202/01/0709, and by the Robert A. Welch Foundation.

-
- [1] C. Rolfs and W. S. Rodney, *Cauldrons in the Cosmos* (University of Chicago Press, Chicago, 1988).
- [2] R. K. Ulrich, in *Essays in Nuclear Astrophysics*, edited by C. A. Barnes, D. D. Clayton, and D. N. Schramm (Cambridge University Press, Cambridge, England, 1982), p. 301.
- [3] M. Lugaro and F. Herwig, Nucl. Phys. **A688**, 201c (2001).
- [4] J.D. King, R.E. Azuma, J.B. Vise, J. Görres, C. Rolfs, H.P. Trautvetter, and A.E. Vlieks, Nucl. Phys. **A567**, 354 (1994).
- [5] G.R. Caughlan and W.A. Fowler, At. Data Nucl. Data Tables **40**, 283 (1988).
- [6] F. Ajzenberg-Selove, Nucl. Phys. **A449**, 1 (1986).
- [7] L. Trache, A. Azhari, H.L. Clark, C.A. Gagliardi, Y.-W. Lui, A.M. Mukhamedzhanov, R.E. Tribble, and F. Carstoiu, Phys. Rev. C **58**, 2715 (1998).
- [8] P. Bem, V. Burjan, V. Kroha, J. Novak, S. Piskor, E. Simcckova, J. Vincour, C.A. Gagliardi, A.M. Mukhamedzhanov, and R.E. Tribble, Phys. Rev. C **62**, 024320 (2000).
- [9] P. Bem *et al.* (private communication).
- [10] C. Angulo *et al.*, Nucl. Phys. **A656**, 3 (1999).