Anisotropic transport properties of ferromagnetic-superconducting bilayers

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We study the transport properties of vortex matter in a superconducting thin film separated by a thin insulator layer from a ferromagnetic layer. We assume an alternating stripe structure for both FM and superconducting (SC) layers as found in S. Erdin *et al.* [Phys. Rev. Lett. **88**, 017001 (2002)]. We calculate the periodic pinning force in the stripe structure resulting from a highly inhomogeneous distribution of the vortices and antivortices. We show that the transport in SC-FM bilayer is highly anisotropic. In the absence of random pinning it displays a finite resistance for the current perpendicular to stripe and is superconducting for the current parallel to stripes. The average vortex velocity, electric field due to the vortex motion, Josephson frequency, and higher harmonics of the vortex oscillatory motion are calculated.

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The interest in heterogeneous ferromagnetic-superconducting systems has grown rapidly in recent years. This interest stems not only from their possible technological applications but also from new physical phenomena arising from the interaction between two order parameters. Typically such a system consists of a superconductor (SC) placed in close proximity with a periodic ferromagnetic structure (FS) such as an array of ferromagnetic dots or holes. The two systems are separated by an infinitely thin layer of insulator oxide that guarantees the suppression of proximity effects. It was demonstrated experimentally[1–3] that the interaction between the superconductor and the ferromagnet may lead to formation of superconducting vortices interacting with the FM film. Theoretical studies of such systems have been done in Refs. 4–7.

Recently, Erdin *et al.*[7] studied the equilibrium structure of a FM-SC bilayer (FSB). They have proved that it represents a two-dimensional periodic stripe domain structure consisting of two equivalent sublattices, in which both the magnetization $m_z(\mathbf{r})$ and the vortex density $n_v(\mathbf{r})$ alternate. Thus, they predicted spontaneous violation of the translational and rotational symmetry in the bilayer. In this Brief Report we study the transport properties of the FSB. They are associated with the driving force acting on the vortex lattice from an external electric current. We show that the FSB exhibits strong anisotropy of the transport properties: the bilayer may be superconducting for the current parallel to the domain walls and resistive when the current is perpendicular to them.

The force acting on a vortex from other vortices which determines the value of critical current can be characterized as the periodic pinning. An extensive development of theory and experiment related to the pinning and its influence on transport in superconductors was discussed in an exhausting review by Blatter *et al.*[8] Our work differs from the studies considered in this review by two features. First, in the preceding works the magnetic field was assumed to be constant in space, whereas in our problem the average magnetic field is zero, it is strongly inhomogeneous in space. Therefore, in our system equal numbers of vortices and antivortices participate in the motion. Second, in these works the pinning force was assumed to be random, whereas in our case the dominant pinning forces are periodic and regular. Martinoli *et al.*[9] created artificially periodic pinning barriers in superconducting films modulating their thickness periodically. The main difference of their modulated structure from one considered in our work is that the domains in the FSB are not confined to the crystal lattice and can move together with the vortices.

Periodic pinning forces in the direction parallel to the stripes do not appear in continuously distributed vortices, their reappearance is associated with the discreteness of the vortex lattice. Therefore, we need to modify the theory[7] to incorporate the discreteness effects. Let us assume that the saturation magnetization per unit area of the FM film is m and its width is L. The energy necessary to create a single Pearl vortex[10] in the superconductor is $\epsilon_{v0}=\epsilon_0\ln(\lambda/\xi)$ with $\epsilon_0=\phi_0^2/16\pi^2\lambda$, where ϕ_0 is the flux quantum, $\lambda=\lambda_L^2/d_s$ is the effective penetration depth,[11] $\lambda_L=\sqrt{m_ec^2/4\pi n_se^2}$ is the London penetration depth, d_s is the thickness of the superconducting layer, and ξ is its coherence length. It was shown in Ref. 7 that the interaction between the superconducting vortices and the magnetization in the stripe structure renormalizes the single-vortex energy to the value $\widetilde{\epsilon_v}=\epsilon_{v0}-m\phi_0$ which must be negative to allow development of the stripes. The density of the superconducting vortices increases when approaching the domain walls and can be expressed as $n_v(x)=(\pi\tilde{m}/L\phi_0)[1/\sin(\pi x/L)]$, where $\tilde{m}=m-(\epsilon_{v0}/\phi_0)$ is the renormalized magnetization of the FM stripe. The vortices spontaneously appear in the superconductor. We assume that the vortices inside one stripe are arranged in parallel chains (Fig. 1). Each chain is periodic with the same lattice constant b along the chain, whereas the distance between kth and $(k+1)$th chain a_k depends on k. The correspondence between this discrete arrangement and continuous approximation[7] is established by the requirement that the local vortex density $n_v(x_k)$ calculated in Ref. 7 must be equal to $(ba_k)^{-1}$. The coordinate x_k is determined in terms of a_k as a sum: $x_k=\sum_{k'=0}^{k-1}a_{k'}$. For definiteness we choose the origin in the center of the stripe. We assume that the total number of the vortex chains $2N$ in a stripe is large. Then some of them are located very close to the domain walls. Let us re-

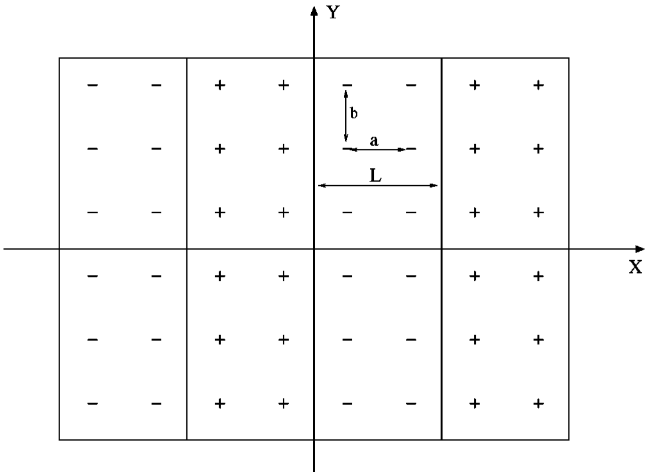

FIG. 1. Schematic vortex distribution in the FM-SC bilayer. The sign $\pm$ refers to the vorticity of the trapped flux.

mind that, in the continuous approximation[7] $n_v = (\pi\tilde{m}/L\phi_0)[1/\sin(\pi x/L)]$, where L is the domain width. Considering the nearest to the domain-wall vortex chain (with the number N), we put $n_v(x_N) = 1/ba_N$. On the other hand $x_N = L - a_N$. Since $a_N/L \ll 1$, we find: $b = \phi_0/\tilde{m}$. The total number of chains in a stripe is $2N$, where $N = b\int_0^{L-\lambda} n_v(x)dx = \frac{1}{2}\ln(L/\lambda)$. We cut the integration (and summation) at a distance $\sim\lambda$ from the domain wall where the continuous approximation breaks. Thus, the minimum value of a is λ. When transport current passes through the superconducting film, the vortices start to move. To simplify the problem we assume that all vortices in each stripe move together as well as all antivortices in the neighboring stripe do. We denote their positions $\mathbf{r}_+ = (x_+, y_+)$ and $\mathbf{r}_- = (x_-, y_-)$, respectively. Forces acting on a moving vortex are the Magnus force, the viscous force, and the periodic pinning force. The Magnus force is $\mathbf{f}_m = \pi n_s \hbar d_s(\mathbf{v}_s - \dot{\mathbf{r}}) \times \hat{z}$, where n_s is the superconducting electron density, v_s is the velocity of the superconducting electron, and $\dot{\mathbf{r}}$ is the vortex velocity. The viscous (friction) force is $\mathbf{f}_f = -\eta\dot{\mathbf{r}}$, where $\eta = \phi_0 H_{c2} d_s/\rho_n c^2$ is the Bardeen-Stephen drag coefficient,[12] H_{c2} is the upper critical magnetic field, ρ_n is the resistivity of the superconducting sample in the normal state, and c is the speed of light. The periodic pinning forces are due to the interaction of the vortex with the pinning centers and the domain walls. In the FM-SC bilayer the pinning force is due to the interaction of the domain walls with the vortices and antivortices and the vortex-vortex interaction U_{vv} given by

$$U_{vv} = \frac{1}{2}\int\int n_v(\mathbf{r})V(\mathbf{r}-\mathbf{r}')n_v(\mathbf{r}')d^2\mathbf{r}d^2\mathbf{r}', \qquad (1)$$

where $V(\mathbf{r}-\mathbf{r}')$ is the pair interaction between vortex located at $\mathbf{r}$ and another at $\mathbf{r}'$. When $|\mathbf{r}-\mathbf{r}'| \gg \lambda$ the pair interaction can be written as $V(\mathbf{r}-\mathbf{r}') = (\phi_0^2/4\pi^2)(1/|\mathbf{r}-\mathbf{r}'|)$.[13] The interaction energy between two parallel chains located at x_l and $x_{l'}$ and vertically shifted with respect to each other by an interval bs $(s \ll 1)$ reads

$$U(x_l, x_{l'}, s) = \sum_{n,m=1}^{N_0} \frac{\frac{\phi_0^2}{8\pi^2}}{\sqrt{(x_l - x_{l'})^2 + (n-m-s)^2 b^2}}, \qquad (2)$$

where N_0 is the number of vortices or antivortices in a single chain. For infinite chains $(N_0 \to \infty)$ Eq. (2) can be rewritten as

$$U(x_l, x_{l'}, s) = \sum_{k=-\infty}^{\infty} \frac{\frac{N_0\phi_0^2}{8\pi^2}}{\sqrt{(x_l - x_{l'})^2 + (k-s)^2 b^2}}. \qquad (3)$$

The sum in Eq. (3) can be calculated using Poisson summation formula.[14,15] Since the force is zero in the continuous approximation, it is possible to retain the lowest nonzero harmonic in the Poisson summation. Thus, we arrive at the following interaction energy of two chains:

$$U(x_l, x_{l'}, s) = \frac{N_0\phi_0^2}{4\pi^2 b}\cos(2\pi s)\chi_{ll'}, \qquad (4)$$

where $\chi_{ll'} = e^{-2\pi(|x_l - x_{l'}|/b)}$. The distance between two chains $|x_l - x_{l'}|$ is larger or equals λ hence $\chi_{ll'} \sim \chi = e^{-(2\pi\lambda/b)}$. Typical value of $\chi_{ll'}$ is $e^{-(\delta_m/4\pi)}$, where $\delta_m = m\phi_0/\epsilon_0 = gS(n_m d_m/n_s d_s)$, with g is Lande factor, S is the ferromagnet elementary spin, n_m and d_m are the electrons density and thickness of the magnetic film, respectively.

We conclude that the amplitude of the periodic potential for displacements parallel to the domains in units of ϵ_0 is exponentially small near the transition temperature. Relative displacements in perpendicular direction have energy barrier $\sim\epsilon_0$ even in continuous approximation. We model the restoring forces by simple sines dependencies $f_x = f_\perp \sin[(2\pi/a)(x_+ - x_-)]$, $f_y = f_{||}\sin[(2\pi/b)(y_+ - y_-)]$, where $f_\perp \sim \epsilon_0/a$ and $f_{||} \sim (\epsilon_0/b)e^{-(\delta_m/4\pi)} \ll f_\perp$.

When the supercurrent is perpendicular to domains, the equations of motion for a vortex and antivortex are

$$\eta\dot{y}_+ = F - \frac{F}{v_s}\dot{x}_+ - f_{||}\sin\left(\frac{2\pi}{b}(y_+ - y_-)\right), \qquad (5)$$

$$\eta\dot{x}_+ = \frac{F}{v_s}\dot{y}_+ + f_\perp\sin\left(\frac{2\pi}{a}(x_+ - x_-)\right), \qquad (6)$$

$$\eta\dot{y}_- = -F + \frac{F}{v_s}\dot{x}_- + f_{||}\sin\left(\frac{2\pi}{b}(y_+ - y_-)\right), \qquad (7)$$

$$\eta\dot{x}_- = -\frac{F}{v_s}\dot{y}_- - f_\perp\sin\left(\frac{2\pi}{a}(x_+ - x_-)\right), \qquad (8)$$

where $F = \pi n_s \hbar d_s v_s$, $f_\perp = \epsilon_0/a$ and $f_{||} = \chi\epsilon_0/b$. If the current is smaller than a critical value, Eqs. (5)–(8) accept a static solution

$$x_+ = x_- = \frac{Fb}{4\pi\eta v_s}\arcsin\left(\frac{F}{f_{||}}\right), \qquad (9)$$

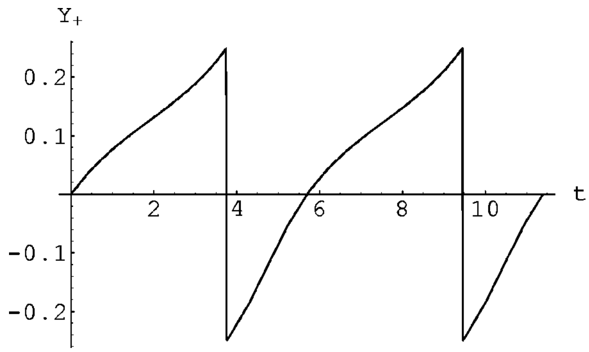

FIG. 2. The vortex displacement as a function of time in the overcritical regime. Time is measured in units of $1/\omega_0^{\perp}$ and y_+ in units of b and $\chi = 10^{-4}$.

$$y_+ = -y_- = \frac{b}{4\pi}\arcsin\left(\frac{F}{f_{||}}\right). \tag{10}$$

It is valid at $F \leqslant f_{||} = \chi\epsilon_0/b$. For $F > f_{||}$ or equivalently, if the current is larger than its critical value, vortices and antivortices start to move. The solution of Eqs. (5)–(8) for $F > F_c$ reads

$$x_+ - x_- = 0, \tag{11}$$

$$x_+ + x_- = \frac{F}{\eta v_s}(y_+ - y_-), \tag{12}$$

$$y_+ - y_- = \frac{b}{\pi}\arctan\left(\frac{f_{||}}{F} + \sqrt{1 - \frac{f_{||}^2}{F^2}}\tan(\omega_0^{\perp} t)\right), \tag{13}$$

$$y_+ + y_- = 0, \tag{14}$$

where $\omega_0^{\perp} = 2\pi\eta v_s^2\sqrt{b^2F^2 - \chi^2\epsilon_0^2}/b^2(F^2 + \eta^2 v_s^2)$ is the Josephson frequency. Thus, the vortices and antivortices acquire the same velocity components $v_{+x} = v_{-x}$ in the direction of the current and opposite velocity components $v_{+y} = -v_{-y}$ in the direction perpendicular to the current. The domain walls do not interfere such a motion if they move in the direction of the current with the same velocity $v_{dw} = v_{+x} = v_{-x}$ as vortices and antivortices. Such a motion is a Goldstone mode. The solution (11)–(13) displays an oscillatory motion of the vortices and antivortices in the direction parallel to the domain walls (Fig. 2), in addition to their motion together with the domain walls along the direction of the current. Higher harmonics of the vortex motion can be calculated analytically. The distribution of vortices (antivortices) is inhomogeneous in the direction perpendicular to the domains. The local electric field $\mathbf{E}$ due to the vortex motion is related to its time-average velocity $\langle \mathbf{v}_+ \rangle$ as $\mathbf{E} = -(q_v\phi_0/c)n_v(\mathbf{r})(\langle \mathbf{v}_+ \rangle \times \hat{\mathbf{z}})$.[8] Therefore, the local field produced by vortices in the direction parallel to the domains is equal but opposite in sign to the one produced by antivortices, while the local field produced by vortices and antivortices in the direction perpendicular to the domains has both equal magnitude and sign. The time average of the vortex (antivortex) velocity over a period $T = 2\pi/\omega_0^{\perp}$ is

$$\langle \mathbf{v}_{+y} \rangle = \pm \frac{\eta\sqrt{F^2 - f_{||}^2}}{\left(\eta^2 + \dfrac{F^2}{v_s^2}\right)}, \tag{15}$$

$$\langle \mathbf{v}_{+x} \rangle = \frac{F\langle v_{+y} \rangle}{\eta v_s}. \tag{16}$$

The time-averaged local-field components are

$$E_x = -\frac{\eta\tilde{m}}{ac}\frac{\sqrt{F^2 - f_{||}^2}}{\left(\dfrac{F^2}{v_s^2} + \eta^2\right)}, \tag{17}$$

$$E_y = \mp\frac{\tilde{m}F}{av_sc}\frac{\sqrt{F^2 - f_{||}^2}}{\left(\dfrac{F^2}{v_s^2} + \eta^2\right)}. \tag{18}$$

The upper sign in Eqs. (15) and (18) refers to the vortices velocity and produced field along the domain while the lower sign refers to those due to antivortices. Nonzero average electric field due to all vortices and antivortices in the FSB appears only in the direction perpendicular to the domains. The critical current J_c is related to F_c as $J_c = (c/\phi_0 d_s)F_c$. Plugging $F_c = \epsilon_0 m\chi/\phi_0$ into the expression for J_c and accepting $\chi = 10^{-4}$–10^{-2}, $b = 10^{-4}$–10^{-5} cm, and $n_s = 10^{22}$ cm^{-3} we find: $J_c \sim 10^3$–10^5 A/cm^2.

When the current flows parallel to the stripes, the FM domain walls stay at rest while vortices and antivortices move both parallel and perpendicular to the domains. The solution of equations of motion for vortices and antivortices shows that they move opposite to one another both in x and y directions. Their motion along x is oscillatory with fundamental frequency $\omega_0^{||} = [2\pi\eta v_s^2\sqrt{a^2F^2 - \epsilon_0^2}/a^2(F^2 + \eta^2 v_s^2)]$. The motion of vortices and antivortices in the parallel direction proceeds until the distance between them becomes half lattice spacing $b/2$. Once the vertical shift between the vortices and antivortices reaches $b/2$, their motion freezes. The critical current in this case is $J_c = n_s\mu_B/2a$, the lattice spacing a is of the order of $\lambda \sim 10^{-5}$–10^{-4} cm, hence the critical current J_c is of the order 10^7–10^8A/cm^2, which is at least 10^2 times larger than the critical current for parallel current. Therefore, the system may be superconducting for the current parallel to the stripes and exhibit finite resistance for perpendicular current. The difference in the critical currents for parallel and perpendicular directions is due to the exponential factor χ which is small if $b \ll \lambda$. The anisotropy is pronounced when δ_m is large which can be achieved by using thicker FM layers and decreasing the density of the superconducting electrons. δ_m is temperature dependent and eventually decreases when temperature decreases starting from T_s. However, at the temperature of vortex disappearance $T_v < T_s$ the value $\tilde{m}$ turns into zero and χ again becomes exponentially small. Thus, anisotropy has a minimum between T_v and T_s.

Kopnin and Vinokur[16] considered a collection of superconducting grains with the wash-board pinning potential a model of random pinning. They obtained a similar result for vortex sliding in external magnetic field with a supercurrent applied. In contrast to their work (they considered vortices only), we consider vortices and antivortices in the field of periodic pinning and completely neglect the random pinning.

Let us discuss briefly how the magnetic field generated by supercurrent changes our result. In Ref. 17 it was shown that at sufficiently small critical magnetic field the domains vanish. Therefore, in general, magnetic field suppresses both the anisotropy and periodic pinning at a critical field for which domains disappear. At such critical field only random pinning prevails. However, the total current per unit length is proportional to the thickness of the SC film and can be kept small.

In conclusion, we studied the transport properties of the FM-SC bilayer in a state with stripe domains of alternating magnetization and vorticity. In the absence of a driving force, the vortices and antivortices are arranged in straight chains configuration. The force between two chains of vortices falls off exponentially as a function of the distance separating the chains. We argued that, in the vicinity of the superconducting transition temperature T_s and vortex disappearance temperature T_v, the distances between chains become much larger than the distances between vortices in the same chain. We solved the equations of motion for vortices and antivortices for the driving current direction parallel and perpendicular to the domains. The calculated parallel to domains critical current is much higher than the perpendicular one at least in a vicinity of the two transition temperatures. This strong transport anisotropy is due to the fact that, for the perpendicular current the induced motion is a Goldstone mode specific for a system of mobile domains and vortices. We expect the ratio of the parallel to perpendicular critical current to be in the range 10^2–10^4 close to the superconducting transition temperature T_s and to the vortex disappearance temperature T_v. The anisotropy decreases rapidly when the temperature goes from the ends of this interval reaching its minimum somewhere inside it. The anisotropy can be destroyed by a rather weak magnetic field perpendicular to the bilayer. This anisotropic transport behavior could serve as a diagnostic tool to discover spontaneous topological structures in magnetic-superconducting systems.

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