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## Gauged Six-dimensional Supergravity from Massive Type IIA

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### ABSTRACT

We obtain the complete non-linear Kaluza-Klein ansatz for the reduction of the bosonic sector of massive type IIA supergravity to the Romans  $F(4)$  gauged supergravity in six dimensions. The latter arises as a consistent warped  $S^4$  reduction.

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The conjectured duality between supergravity on an anti-de Sitter (AdS) background and a superconformal field theory (CFT) on its boundary [1, 2, 3, 4] has led to a renewed interest in the mechanism whereby the relevant gauged supergravities can be obtained by Kaluza-Klein reduction from higher dimensions. It has long been known that the maximal gauged theories in  $D = 4$  and  $D = 7$  can be obtained by reduction of eleven-dimensional supergravity on  $S^7$  or  $S^4$ , and that the maximal gauged theory in  $D = 5$  can be obtained from an  $S^5$  reduction from type IIB supergravity. In each case, it is believed that the reduction is consistent, in the sense that the reduction ansatz, with its truncation to the fields of the supergravity multiplet, satisfies the higher-dimensional equations of motion provided that the lower-dimensional equations of motion are satisfied. This is important in the context of the AdS/CFT correspondence, since it implies that massive fields can be ignored when calculating correlation functions in the conformal field theory [3].

A long-standing puzzle has been to obtain a higher-dimensional Kaluza-Klein interpretation for the gauged supergravity theory in six dimensions [5], whose dual description on its boundary is a five-dimensional  $N = 2$  superconformal field theory [6, 7]. It was suggested in [8] that it could be related to the ten-dimensional massive type IIA theory [9]. Recently, it was shown that the massive IIA theory admitted an  $\text{AdS}_6 \times S^4$  solution, with a “warped-product” metric [10]. This was derived as the near-horizon limit of a localised D4-D8 brane configuration [11].

In this letter we resolve the puzzle, by obtaining the complete non-linear bosonic Kaluza-Klein ansatz for the reduction of the massive IIA theory on  $S^4$ , and showing that it gives a consistent truncation to the six-dimensional gauged theory. Our starting point is the bosonic Lagrangian of massive type IIA supergravity [9]. In the language of differential forms, it is given by [12]

$$\begin{aligned} \mathcal{L}_{10} = & \hat{R} \hat{*} \mathbf{1} - \frac{1}{2} \hat{*} d\hat{\phi} \wedge d\hat{\phi} - \frac{1}{2} e^{\frac{3}{2}\hat{\phi}} \hat{*} \hat{F}_{(2)} \wedge \hat{F}_{(2)} - \frac{1}{2} e^{-\hat{\phi}} \hat{*} \hat{F}_{(3)} \wedge \hat{F}_{(3)} - \frac{1}{2} e^{\frac{1}{2}\hat{\phi}} \hat{*} \hat{F}_{(4)} \wedge \hat{F}_{(4)} \\ & - \frac{1}{2} d\hat{A}_{(3)} \wedge d\hat{A}_{(3)} \wedge \hat{A}_{(2)} - \frac{1}{6} m d\hat{A}_{(3)} \wedge (\hat{A}_{(2)})^3 - \frac{1}{40} m^2 (\hat{A}_{(2)})^5 - \frac{1}{2} m^2 e^{\frac{5}{2}\hat{\phi}} \hat{*} \mathbf{1}, \end{aligned} \quad (1)$$

where the field strengths are given in terms of potentials by

$$\begin{aligned} \hat{F}_{(2)} &= d\hat{A}_{(1)} + m \hat{A}_{(2)}, & \hat{F}_{(3)} &= d\hat{A}_{(2)}, \\ \hat{F}_{(4)} &= d\hat{A}_{(3)} + \hat{A}_{(1)} \wedge d\hat{A}_{(2)} + \frac{1}{2} m \hat{A}_{(2)} \wedge \hat{A}_{(2)}. \end{aligned} \quad (2)$$

In the above, we have used the hat symbol to denote the ten-dimensional fields and Hodge dual, and subscripts on form fields indicate the degrees of the forms. It follows that the equations of motion and Bianchi identities are

$$d(e^{\frac{1}{2}\hat{\phi}} \hat{*} \hat{F}_{(4)}) = -\hat{F}_{(3)} \wedge \hat{F}_{(4)}, \quad d(e^{\frac{3}{2}\hat{\phi}} \hat{*} \hat{F}_{(2)}) = -e^{\frac{1}{2}\hat{\phi}} \hat{*} \hat{F}_{(4)} \wedge \hat{F}_{(3)},$$

$$\begin{aligned}
d(e^{-\hat{\phi}} \hat{*}\hat{F}_{(3)}) &= -\frac{1}{2}\hat{F}_{(4)} \wedge \hat{F}_{(4)} - m e^{\frac{3}{2}\hat{\phi}} \hat{*}\hat{F}_{(2)} - e^{\frac{1}{2}\hat{\phi}} \hat{*}\hat{F}_{(4)} \wedge \hat{F}_{(2)}, \\
d\hat{*}d\hat{\phi} &= -\frac{5}{4}m^2 e^{\frac{5}{2}\hat{\phi}} \hat{*}\mathbf{1} - \frac{3}{4}e^{\frac{3}{2}\hat{\phi}} \hat{*}\hat{F}_{(2)} \wedge F_{(2)} + \frac{1}{2}e^{-\hat{\phi}} \hat{*}\hat{F}_{(3)} \wedge \hat{F}_{(3)} - \frac{1}{4}e^{\frac{1}{2}\hat{\phi}} \hat{*}\hat{F}_{(4)} \wedge \hat{F}_{(4)}, \\
d\hat{F}_{(4)} &= \hat{F}_{(2)} \wedge \hat{F}_{(3)}, \quad d\hat{F}_{(3)} = 0, \quad d\hat{F}_{(2)} = m \hat{F}_{(3)},
\end{aligned} \tag{3}$$

for the form fields and dilaton, together with the Einstein equation (in vielbein components)

$$\begin{aligned}
\hat{R}_{AB} &= \frac{1}{2}\partial_A \hat{\phi} \partial_B \hat{\phi} + \frac{1}{16}m^2 e^{\frac{5}{2}\hat{\phi}} \eta_{AB} + \frac{1}{12}e^{\frac{1}{2}\hat{\phi}} \left( \hat{F}_{(4)AB}^2 - \frac{3}{32}\hat{F}_{(4)}^2 \eta_{AB} \right) \\
&+ \frac{1}{4}e^{-\hat{\phi}} \left( \hat{F}_{(3)AB}^2 - \frac{1}{12}\hat{F}_{(3)}^2 \eta_{AB} \right) + \frac{1}{2}e^{\frac{3}{2}\hat{\phi}} \left( \hat{F}_{(2)AB}^2 - \frac{1}{16}\hat{F}_{(2)}^2 \eta_{AB} \right).
\end{aligned} \tag{4}$$

We shall now describe how we can perform a 4-sphere reduction of the massive IIA theory, with a consistent truncation to the fields of gauged  $N = 1$ ,  $D = 6$  supergravity. The bosonic fields in this theory comprise the metric, a dilaton, a 2-form potential and a 1-form potential, together with the gauge potentials of  $SU(2)$  Yang-Mills. The bosonic Lagrangian [5], converted to the language of differential forms, is

$$\begin{aligned}
\mathcal{L}_6 &= R * \mathbf{1} - \frac{1}{2} * d\phi \wedge d\phi - g^2 \left( \frac{2}{9} e^{\frac{3}{2}\phi} - \frac{8}{3} e^{\frac{1}{2}\phi} - 2e^{-\frac{1}{2}\phi} \right) * \mathbf{1} \\
&- \frac{1}{2} e^{-\sqrt{2}\phi} * F_{(3)} \wedge F_{(3)} - \frac{1}{2} e^{\frac{1}{\sqrt{2}}\phi} \left( * F_{(2)} \wedge F_{(2)} + * F_{(2)}^i \wedge F_{(2)}^i \right) \\
&- A_{(2)} \wedge \left( \frac{1}{2} dA_{(1)} \wedge dA_{(1)} + \frac{1}{3} g A_{(2)} \wedge dA_{(1)} + \frac{2}{27} g^2 A_{(2)} \wedge A_{(2)} + \frac{1}{2} F_{(2)}^i \wedge F_{(2)}^i \right),
\end{aligned} \tag{5}$$

where  $F_{(3)} = dA_{(2)}$ ,  $F_{(2)} = dA_{(1)} + \frac{2}{3}g A_{(2)}$  and  $F_{(2)}^i = dA_{(1)}^i + \frac{1}{2}g \epsilon_{ijk} A_{(1)}^j \wedge A_{(1)}^k$ . Here  $*$  is the six-dimensional Hodge dual.

We find that the reduction ansätze for the metric, form fields and dilaton of the ten-dimensional massive type IIA theory are

$$\begin{aligned}
d\hat{s}_{10}^2 &= (\sin \xi)^{\frac{1}{12}} X^{\frac{1}{8}} \left[ \Delta^{\frac{3}{8}} ds_6^2 + 2g^{-2} \Delta^{\frac{3}{8}} X^2 d\xi^2 + \frac{1}{2}g^{-2} \Delta^{-\frac{5}{8}} X^{-1} \cos^2 \xi \sum_{i=1}^3 (\sigma^i - g A_{(1)}^i)^2 \right], \\
\hat{F}_{(4)} &= -\frac{\sqrt{2}}{6} g^{-3} s^{1/3} c^3 \Delta^{-2} U d\xi \wedge \epsilon_{(3)} - \sqrt{2} g^{-3} s^{4/3} c^4 \Delta^{-2} X^{-3} dX \wedge \epsilon_{(3)} \\
&- \sqrt{2} g^{-1} s^{1/3} c X^4 * F_{(3)} \wedge d\xi - \frac{1}{\sqrt{2}} s^{4/3} X^{-2} * F_{(2)} \\
&+ \frac{1}{\sqrt{2}} g^{-2} s^{1/3} c F_{(2)}^i h^i \wedge d\xi - \frac{1}{4\sqrt{2}} g^{-2} s^{4/3} c^2 \Delta^{-1} X^{-3} F_{(2)}^i \wedge h^j \wedge h^k \epsilon_{ijk}, \\
\hat{F}_{(3)} &= s^{2/3} F_{(3)} + g^{-1} s^{-1/3} c F_{(2)} \wedge d\xi, \\
\hat{F}_{(2)} &= \frac{1}{\sqrt{2}} s^{2/3} F_{(2)}, \quad e^{\hat{\phi}} = s^{-5/6} \Delta^{1/4} X^{-5/4},
\end{aligned} \tag{6}$$

where  $X$  is related to the dilaton  $\phi$  in (5) by  $X = e^{-\frac{1}{2\sqrt{2}}\phi}$ , and

$$\begin{aligned}
\Delta &\equiv X \cos^2 \xi + X^{-3} \sin^2 \xi, \\
U &\equiv X^{-6} s^2 - 3X^2 c^2 + 4X^{-2} c^2 - 6X^{-2}.
\end{aligned} \tag{7}$$

The quantities  $\sigma^i$  are left-invariant 1-forms on  $S^3$ , which satisfy  $d\sigma^i = -\frac{1}{2}\epsilon_{ijk}\sigma^j \wedge \sigma^k$ . We have also defined  $h^i \equiv \sigma^i - g A_{(1)}^i$ ,  $\epsilon_{(3)} \equiv h^1 \wedge h^2 \wedge h^3$ , and  $s = \sin \xi$  and  $c = \cos \xi$ . The gauge coupling constant  $g$  is related to the mass parameter  $m$  of the massive type IIA theory by  $m = \frac{\sqrt{2}}{3} g$ .

It is useful also to present the expressions for the ten-dimensional Hodge duals of the form fields given above, and for  $d\hat{\phi}$ . We find that they are given by

$$\begin{aligned}
e^{\frac{1}{2}\hat{\phi}} \hat{*}\hat{F}_{(4)} &= -\frac{\sqrt{2}}{3}g U \epsilon_{(6)} + 4\sqrt{2}g^{-1} s c X^{-1} *dX \wedge d\xi \\
&\quad -\frac{\sqrt{2}}{4}g^{-3} c^4 \Delta^{-1} X F_{(3)} \wedge \epsilon_{(3)} + \frac{1}{2\sqrt{2}}g^{-4} s c^3 \Delta^{-1} X^{-3} F_{(2)} \wedge d\xi \wedge \epsilon_{(3)} \\
&\quad -\frac{1}{4\sqrt{2}}g^{-2} c^2 X^{-2} *F_{(2)}^i \wedge h^j \wedge h^k \epsilon_{ijk} + \frac{1}{\sqrt{2}}g^{-2} s c X^{-2} *F_{(2)}^i \wedge h^i \wedge d\xi, \\
e^{-\hat{\phi}} \hat{*}\hat{F}_{(3)} &= \frac{1}{2}g^{-4} s^{5/3} c^3 \Delta^{-1} X *F_{(3)} \wedge d\xi \wedge \epsilon_{(3)} - \frac{1}{4}g^{-3} s^{2/3} c^4 \Delta^{-1} X^{-1} *F_{(2)} \wedge \epsilon_{(3)}, \\
e^{\frac{3}{2}\hat{\phi}} \hat{*}\hat{F}_{(2)} &= \frac{1}{2\sqrt{2}}g^{-4} s^{-1/3} c^3 X^{-2} *F_{(2)} \wedge d\xi \wedge \epsilon_{(3)}, \\
\hat{*}d\hat{\phi} &= -\frac{1}{2}g^{-4} s^{1/3} c^3 (X c^2 + 2X^{-3} s^2) \Delta^{-1} X^{-1} *dX \wedge d\xi \wedge \epsilon_{(3)} \\
&\quad + \frac{1}{16}g^{-2} s^{1/3} c^3 (\Delta^{-1} \partial_\xi \Delta - \frac{10}{3} \cot \xi) X^{-2} \epsilon_{(6)} \wedge \epsilon_{(3)},
\end{aligned} \tag{8}$$

where  $\epsilon_{(6)}$  is the volume form of the metric  $ds_6^2$ .

If we set  $X = 1$  and  $A_{(1)}^i = 0$ , then  $ds_6^2$  becomes an Einstein metric, which can, for example, be  $\text{AdS}_6$ . In this case, the ten-dimensional geometry becomes  $\text{AdS}_6 \times S^4$  with a warp factor [10],

$$ds_{10}^2 = (\sin \xi)^{\frac{1}{12}} \left[ ds_{\text{AdS}_6}^2 + 2g^{-2} \left( d\xi^2 + \frac{1}{4} \cos^2 \xi \sum_i (\sigma^i)^2 \right) \right], \tag{9}$$

which is the near-horizon limit [10] of a localised D4-D8 brane configuration [11].<sup>1</sup> The configuration is a solution of the massive type IIA theory, where the  $\text{AdS}_6$  metric  $ds_{\text{AdS}_6}^2$  has Ricci tensor  $R_{ab} = -\frac{10}{9}g^2 g_{ab}$  and the 4-sphere metric  $2g^{-2} (d\xi^2 + \frac{1}{4} \cos^2 \xi \sum_i (\sigma^i)^2)$  has Ricci tensor  $R_{\alpha\beta} = \frac{3}{2}g^2 g_{\alpha\beta}$ . It follows from the ansatz (6) that the ten-dimensional fields  $\hat{F}_{(4)}$  and  $\hat{\phi}$  take the non-vanishing forms

$$\hat{F}_{(4)} = \frac{5\sqrt{2}}{6} g^{-3} (\sin \xi)^{1/3} \cos^3 \xi d\xi \wedge \epsilon_{(3)}, \quad e^{\hat{\phi}} = (\sin \xi)^{-5/6}. \tag{10}$$

When the fields  $X$  and  $A_{(1)}^i$  are excited,  $X$  parameterises inhomogeneous deformations of the 4-sphere, leaving the foliating 3-spheres intact, while  $A_{(1)}^i$  describes deformations of the 3-spheres corresponding to right translations under  $SU(2)$ .

<sup>1</sup>To be more precise, the  $S^4$  here is not really the entire 4-sphere, but rather just the upper hemisphere of a 4-sphere, viewed as a foliation of 3-spheres [10]. This is because the conformal warp factor  $(\sin \xi)^{1/12}$  approaches zero as the ‘‘latitude’’ coordinate  $\xi$  approaches the equatorial 3-sphere at  $\xi = 0$ , thus defining a boundary to the 4-manifold.

Substituting the ansätze (6) into the equations of motion and Bianchi identities (3) for the form fields and dilaton of the massive type IIA theory, we find that they are satisfied provided the six-dimensional fields satisfy the following equations of motion:

$$\begin{aligned}
d(X^4 *F_{(3)}) &= -\frac{1}{2}F_{(2)} \wedge F_{(2)} - \frac{1}{2}F_{(2)}^i \wedge F_{(2)}^i - \frac{2}{3}g X^{-2} *F_{(2)}, \\
d(X^{-2} *F_{(2)}) &= -F_{(2)} \wedge F_{(3)}, \quad D(X^{-2} *F_{(2)}^i) = -F_{(2)}^i \wedge F_{(3)}, \\
d(X^{-1} *dX) &= \frac{1}{8}X^{-2} (*F_{(2)} \wedge F_{(2)} + *F_{(2)}^i \wedge F_{(2)}^i) - \frac{1}{4}X^4 *F_{(3)} \wedge F_{(3)} \\
&\quad + g^2 \left( \frac{1}{6}X^{-6} - \frac{2}{3}X^{-2} + \frac{1}{2}X^2 \right) *1,
\end{aligned} \tag{11}$$

where  $D$  is the Yang-Mills gauge-covariant exterior derivative,  $D\omega^i = d\omega^i + g \epsilon_{ijk} A_{(1)}^j \wedge \omega^k$ . Note that the Bianchi identities for  $\hat{F}_{(3)}$  and  $\hat{F}_{(2)}$  are satisfied identically, whilst that for  $\hat{F}_{(4)}$  already implies the equations of motion for  $F_{(3)}$  and  $F_{(2)}$ .

Evaluating the ten-dimensional Einstein equation (4) with the ansätze (6) is a more exacting task. After doing so, we find that consistency again requires the equations of motion for  $F_{(2)}^i$  and  $X$  given in (11), and in addition it implies the six-dimensional Einstein equation

$$\begin{aligned}
R_{\mu\nu} &= 4X^{-2} \partial_\mu X \partial_\nu X + g^2 \left( \frac{1}{18}X^{-6} - \frac{2}{3}X^{-2} - \frac{1}{2}X^2 \right) g_{\mu\nu} + \frac{1}{4}X^4 (F_{(3)\mu\nu}^2 - \frac{1}{6}F_{(3)}^2 g_{\mu\nu}) \\
&\quad + \frac{1}{2}X^{-2} (F_{(2)\mu\nu}^2 - \frac{1}{8}F_{(2)}^2 g_{\mu\nu}) + \frac{1}{2}X^{-2} ((F_{(2)}^i)_{\mu\nu}^2 - \frac{1}{8}(F_{(2)}^i)^2 g_{\mu\nu}).
\end{aligned} \tag{12}$$

It is now straightforward to see that the full set of six-dimensional equations of motion (11) and (12) are precisely those which follow from the Lagrangian (5) for  $N = 1$ ,  $D = 6$  gauged supergravity.

In our derivation, the consistency of the reduction ansatz is definitively established, since we have explicitly substituted it into the higher-dimensional equations of motion, and shown that these equations are satisfied if and only if the lower-dimensional equations of motion are satisfied. This is, by definition, what one means by a consistent Kaluza-Klein reduction. The Kaluza-Klein reduction procedure is sometimes stated instead at the level of the action; namely, that one would substitute the ansatz into the higher-dimensional action, integrate over the internal directions, and thereby arrive at an action for the lower-dimensional fields. Of course in such an approach, it would be necessary to construct an independent argument for why the reduction ansatz was a consistent one. However, there are other reasons also why substituting the ansatz into the Lagrangian might be problematical. To illustrate this, it is instructive to look at the reduction we have considered in this letter, simplified initially by restricting the fields to just the metric and the dilaton.

In the gravity-scalar sector, the ansatz for the field strengths can be rewritten in terms of the potentials, since we can then write  $\hat{A}_{(3)} = \frac{1}{4\sqrt{2}}g^{-3} s^{4/3} (3 + 2c^2 \Delta^{-1} X^{-3}) \epsilon_{(3)}$ , as may

be seen from (6). Substituting this and the other non-zero ansätze into the ten-dimensional Lagrangian (1) gives  $\mathcal{L}_{10} = \frac{1}{2}g^{-4} s^{1/3} c^3 (R - \frac{1}{2}(\partial\phi)^2 + W) \sqrt{-g_6}$ , where

$$W = -\frac{1}{36}g^2 s^{-2} \Delta^{-2} X^{-12} \left( 8s^2 + 6s^4(1 - 27s^2)X^4 + 6s^2(4 - 45s^2 + 38s^4)X^8 - c^2(1 + 118s^2 - 2s^4)X^{12} - 72s^2 c^4 X^{16} \right). \quad (13)$$

Although the  $R$  and  $(\partial\phi)^2$  terms have a uniform  $\xi$ -dependent prefactor, the term  $W$ , associated with the scalar potential, does not. Integration over the internal coordinate  $\xi$  does not really make sense, since there is a divergence at  $\xi = 0$ . One can make a suitable regularisation and thereby obtain the scalar potential as given in (5), but this is unsatisfactory since the result is scheme-dependent.<sup>2</sup> Moreover, when the higher-degree fields of the six-dimensional theory are included, it is no longer possible to rewrite the ansatz for  $\hat{F}_{(4)}$  in (6) as an ansatz for  $\hat{A}_{(3)}$ . We thus expect in this case that one would not be able to obtain the six-dimensional Lagrangian (5) by substituting the ansätze into the ten-dimensional one. It should be emphasised, however, that this is not a drawback in the reduction procedure; rather, it just serves to illustrate that Kaluza-Klein reduction is in general rather more subtle than in the simple case of toroidal reduction. The key point is that given a consistent reduction, one has a way of embedding solutions of the lower-dimensional equations of motion as solutions of the higher-dimensional ones.

An example of such a six-dimensional solution is an AdS black hole, supported by a single component of the  $SU(2)$  Yang-Mills fields. We find that the solution is given by

$$\begin{aligned} ds_6^2 &= -H^{-3/2} f dt^2 + H^{1/2} (f^{-1} dr^2 + r^2 d\Omega_{4,k}^2), \\ \phi &= \frac{1}{\sqrt{2}} \log H, \quad A_{(1)}^3 = \sqrt{2k}(1 - H^{-1}) \coth \beta dt, \\ f &= k - \frac{\mu}{r^3} + \frac{2}{9}g^2 r^2 H^2, \quad H = 1 + \frac{\mu \sinh^2 \beta}{k r^3}, \end{aligned}$$

where we have, for definiteness, chosen to use the  $i = 3$  component of the Yang-Mills fields  $A_{(1)}^i$ . Another example is a supersymmetric domain wall, supported by the scalar potential [13]. It is straightforward to oxidise these solutions to ten dimensions, using our ansätze (6).

If the parameter  $\mu$  is set to zero in the AdS black-hole solution, the six-dimensional metric

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<sup>2</sup>The occurrence of the divergence is associated with the fact that the metric in (6) has the warp factor  $(\sin \xi)^{1/12}$ , which vanishes at  $\xi = 0$ . This singular behaviour is an inherent feature of the massive type IIA theory, resulting from the scalar potential  $e^{\frac{5}{2}\hat{\phi}}$ , which has no stationary point. (Since the dilaton also diverges as  $\xi$  approaches zero, implying a passage to the strong-coupling regime of the type I string theory, the effective supergravity will in any case receive modifications.) An analogous calculation in the  $S^4$  reduction of eleven-dimensional supergravity, where no  $\xi$ -dependent warp factor arises, is free from any singular behaviour [16].

becomes simply  $\text{AdS}_6$ , and, as we remarked previously, the oxidation to  $D = 10$  gives the near-horizon limit of the localised D4-D8 brane configuration (in the case  $k = 0$ ). When  $\mu$  is instead non-zero, we expect that the ten-dimensional interpretation will be that the D4-D8 brane system will acquire a rotation, with angular frequency equal to the black-hole charge, analogous to the cases discussed in [14, 15].

Another six-dimensional solution is the non-supersymmetric  $\text{AdS}_6$  [5], corresponding to the second stationary point  $X = 3^{-1/4}$  of the potential. It is interesting to note that the factor  $\Delta$  appearing in the metric ansatz (6), which takes the value  $\Delta = 1$  in the  $X = 1$  supersymmetric  $\text{AdS}_6$  solution, now takes the form  $\Delta = 3^{-1/4}(1 + 2\sin^2\xi)$ , implying a distortion of the 4-sphere.

To summarise, we have derived the gauged six-dimensional supergravity by performing a consistent Kaluza-Klein reduction of massive type IIA supergravity. (For the sake of simplicity, we concentrated on the full bosonic sector of the theories; the fermionic sector will be addressed elsewhere.) The metric ansatz describes a warped product of the six-dimensional spacetime and a 4-sphere. The warp factor depends on the latitude coordinate of the 4-sphere, viewed as a foliation of 3-spheres. Since it vanishes on the equator, the geometry of the internal space is really the upper hemisphere of the 4-sphere, with the equator as boundary. (This is the region where type I string theory becomes strongly coupled, and on the dual weakly coupled heterotic string theory side a gauge enhancement takes place.) We presented examples of six-dimensional solutions that can now be re-interpreted as solutions of the massive IIA theory. More generally, our construction opens the door to the higher-dimensional re-interpretation of any solution of the six-dimensional theory, including, for example, non-abelian configurations.

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