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Veneziano Amplitude for Winding Strings †

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String configurations with nonzero winding number describe soliton string states. We compute the Veneziano amplitude for the scattering of arbitrary winding states and show that in the large radius limit the strings always scatter trivially and with no change in the individual winding numbers of the strings. In this limit, then, these states scatter as true solitons.

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In recent work [1], we obtained dynamical evidence for the identification of the Dabholkar string soliton [2,3] with the underlying fundamental string by comparing the scattering of these soliton solutions with expectations from a Veneziano amplitude computation for macroscopic fundamental strings. These latter states were represented as $n = 1$ winding states in the large winding radius limit. A computation of the dynamical force between two identical strings [4] and of the metric on moduli space for the scattering of two string solitons [5] both yielded the result of trivial scattering, in agreement with the Veneziano amplitude calculation.

In this paper we generalize the $n = 1$ Veneziano amplitude result to arbitrary incoming winding states. In particular, we find that the Veneziano amplitude vanishes in the large radius limit except when the final winding numbers are identical to the initial ones and the scattering angle is zero. In other words, for arbitrary winding number, these states scatter as true solitons.

The scattering problem is set up in four dimensions, as the kinematics correspond essentially to a four dimensional scattering problem, and strings in higher dimensions generically miss each other anyway [6]. The precise compactification scheme is irrelevant to our purposes.

The winding state strings then reside in four spacetime dimensions (0123), with one of the dimensions, say x_3, taken to be periodic with period R, called the winding radius. The winding number n describes the number of times the string wraps around the winding dimension:

$$x_3 \equiv x_3 + 2\pi R n, \tag{1}$$

and the length of the string is given by $L = nR$. The integer m, called the momentum number of the winding configuration, labels the allowed momentum eigenvalues. The momentum in the winding direction is thus given by

$$p^3 = \frac{m}{R}. \tag{2}$$

The number m is restricted to be an integer so that the quantum wave function $e^{ip\cdot x}$ is single valued. The total momentum of each string can be written as the sum of a right momentum and a left momentum

$$p^\mu = p_R^\mu + p_L^\mu, \tag{3}$$

where $p^\mu_{R,L} = (E, E\vec{v}, \frac{m}{2R} \pm nR)$, $\vec{v}$ is the transverse velocity and R is the winding radius. The mode expansion of the general configuration $X(\sigma,\tau)$ in the winding direction satisfying the two-dimensional wave equation and the closed string boundary conditions can be written as the sum of right moving pieces and left moving pieces, each with the mode expansion of an open string[7]

$$\begin{aligned} X(\sigma,\tau) &= X_R(\tau-\sigma) + X_L(\tau+\sigma) \\ X_R(\tau-\sigma) &= x_R + p_R(\tau-\sigma) + \frac{i}{2}\sum_{n=0}\frac{1}{n}\alpha_n e^{-2in(\tau-\sigma)} \\ X_L(\tau+\sigma) &= x_L + p_L(\tau+\sigma) + \frac{i}{2}\sum_{n=0}\frac{1}{n}\tilde{\alpha}_n e^{-2in(\tau+\sigma)}. \end{aligned} \tag{4}$$

The right moving and left moving components are then essentially independent parts with corresponding vertex operators, number operators and Virasoro conditions.

The winding configuration represented by $X(\sigma,\tau)$ describes a soliton string state. It is therefore a natural choice to compare the dynamics of these states with the Dabholkar string solitons in order to determine whether we can identify the solutions of the supergravity field equations with infinitely long fundamental strings. Accordingly, we compared the scattering of $n=1$ winding states in the limit of large winding radius with the scattering of the Dabholkar solitons and in both situations found trivial scattering of identical strings [1]. In this paper, we consider the dynamics of arbitrary winding states.

Our kinematic setup is as follows. We consider the scattering of two straight macroscopic strings in the CM frame with winding numbers n_1 and n_2 and momentum number $\pm m$. The incoming momenta in the CM frame are given by

$$\begin{aligned} p^\mu_{1R,L} &= (E_1, E_1\vec{v}_1, \frac{m}{2R} \pm n_1 R) \\ p^\mu_{2R,L} &= (E_2, E_2\vec{v}_2, -\frac{m}{2R} \pm n_2 R). \end{aligned} \tag{5}$$

The outgoing momenta (with momentum number $\pm m'$) are given by

$$\begin{aligned} -p^\mu_{3R,L} &= (E_3, E_3\vec{w}_3, \frac{m'}{2R} \pm n_3 R) \\ -p^\mu_{4R,L} &= (E_4, E_4\vec{w}_4, -\frac{m'}{2R} \pm n_4 R), \end{aligned} \tag{6}$$

where from conservation of momentum and winding number we have

$$\begin{aligned} & E_1 + E_2 = E_3 + E_4 \\ & E_1\vec{v}_1 + E_2\vec{v}_2 = 0 \\ & E_3\vec{w}_3 + E_4\vec{w}_4 = 0 \\ & n_1 + n_2 = n_3 + n_4 \end{aligned} \tag{7}$$

and where $\vec{v}_i, i = 1, 2$ and $\vec{w}_k, k = 3, 4$ are the incoming and outgoing velocities of the strings in the transverse $x - y$ plane. For simplicity, assume $\vec{v}_1 = v_1\hat{x}$ and $\vec{w}_3 = w_3(\cos\theta\hat{x} + \sin\theta\hat{y})$. For now we assume no longitudinal excitation ($m = m'$), but it turns out that our analysis is unaffected by the possibility of excitation. In the large R limit, the transition amplitude for arbitrary longitudinal excitation is dominated by a factor which decays exponentially with R. Following the same counting argument as in [1], one can show that the number of possible excited transitions is bounded by a polynomial in R. Thus the total amplitude is dominated by the exponential factor and it is therefore sufficient to consider the $m = m'$ case in this limit.

As usual, the Virasoro conditions $L_0 = \widetilde{L}_0 = 1$ must hold, where

$$\begin{aligned} L_0 &= N + \tfrac{1}{2}(p_R^\mu)^2 \\ \widetilde{L}_0 &= \widetilde{N} + \tfrac{1}{2}(p_L^\mu)^2 \end{aligned} \tag{8}$$

are the Virasoro operators[7] and where N and $\widetilde{N}$ are the number operators for the right- and left-moving modes respectively:

$$\begin{aligned} N &= \sum \alpha_{-n}^\mu \alpha_{n\mu} \\ \widetilde{N} &= \sum \tilde{\alpha}_{-n}^\mu \tilde{\alpha}_{n\mu}, \end{aligned} \tag{9}$$

where we sum over all dimensions, including the compactified ones. It follows from the Virasoro conditions that

$$\begin{aligned} \widetilde{N} - N &= mn \\ E^2(1 - v^2) &= 2N - 2 + \left(\frac{m}{2R} + nR\right)^2. \end{aligned} \tag{10}$$

We begin with a computation of the scattering of identical $n = 1$ states but with arbitrary final winding states (with total winding number adding up to 2). To that end, we set $n_1 = n_2 = 1$ and consider for simplicity the scattering of tachyonic winding states. For our purposes, the nature of the string winding states considered is irrelevant. A similar calculation for massless bosonic strings or heterotic strings, for example, will be slightly more complicated (involving kinematic factors), but will nevertheless exhibit the same essential behaviour in the large radius limit (i.e. exponential decay). For tachyonic winding states we have $N = \widetilde{N} = m = 0$. The kinematic setup reduces to

$$\begin{aligned} p_{1R,L}^\mu &= (E, E\vec{v}, \pm R) \\ p_{2R,L}^\mu &= (E, -E\vec{v}, \pm R) \\ -p_{3R,L}^\mu &= (E', E'\vec{w}, \pm nR) \\ -p_{4R,L}^\mu &= (2E - E', -E'\vec{w}, \pm(2 - n)R), \end{aligned} \tag{11}$$

where conservation of momentum and winding number have been used. (10) reduces to

$$\begin{aligned} E^2(1-v^2) &= R^2 - 2 \\ E'^2(1-w^2) &= n^2R^2 - 2, \end{aligned} \tag{12}$$

with

$$E' = E + \frac{(n-1)R^2}{E}. \tag{13}$$

In the standard computation of the four point function for closed string tachyons, we rely on the independence of the right and left moving open strings. For the tachyonic winding state, we also separate the right and left movers with vertex operators given by $V_R = e^{ip_R \cdot x_R}$ and $V_L = e^{ip_L \cdot x_L}$ respectively to arrive at the following expression for the amplitude

$$A_4 = \frac{\kappa^2}{4} \int d\mu_4(z) \prod_{i<j} |z_i - z_j|^{p_{iR}\cdot p_{jR}} |z_i - z_j|^{p_{iL}\cdot p_{jL}}. \tag{14}$$

Since it easily follows from our kinematic setup that $p_{iR} \cdot p_{jR} = p_{iL} \cdot p_{jL}$ holds for this configuration, the tree level 4-point function reduces to the usual Veneziano amplitude for closed tachyonic strings[6]

$$\begin{aligned} A_4 &= \frac{\kappa^2}{4} B(-1-s/2, -1-t/2, -1-u/2) \\ &= (\frac{\kappa^2}{4}) \frac{\Gamma(-1-s/2)\Gamma(-1-t/2)\Gamma(-1-u/2)}{\Gamma(2+s/2)\Gamma(2+t/2)\Gamma(2+u/2)}, \end{aligned} \tag{15}$$

where the Mandelstam variables (s,t,u) are identical for right and left movers and are given by

$$\begin{aligned} s &= 4(E^2 - R^2) \\ t &= -2EE'(1 + vw\cos\theta) + 2nR^2 - 4 \\ u &= -2EE'(1 - vw\cos\theta) + 2nR^2 - 4. \end{aligned} \tag{16}$$

A quick check using (12) and (13) shows that $s + t + u = -8$. For $n = 1$, we recover the case considered in [1]. There we showed that $A_4 \to 0$ as $R \to \infty$ except for at the poles at $\theta = 0, \pi$, corresponding to trivial scattering for identical bosonic states. We use the identity $\Gamma(1-a)\Gamma(a)\sin\pi a = \pi$ to rewrite A_4 as

$$A_4 = (\frac{\kappa^2}{4\pi}) \left[\frac{\Gamma(-1-t/2)\Gamma(-1-u/2)}{\Gamma(2+s/2)} \right]^2 \left(\frac{\sin(-\pi t/2)\sin(-\pi u/2)}{\sin \pi s/2} \right). \tag{17}$$

The sinusoidal factor contains the usual s-channel poles. From the Stirling approximation $\Gamma(u) \sim \sqrt{2\pi} u^{u-1/2} e^{-u}$ for large u, we obtain in the limit $R \to \infty$

$$A_4 \sim \left[\frac{\alpha^\alpha \beta^\beta}{\gamma^\gamma}\right]^2 \left(\frac{\sin(-\pi t/2)\sin(-\pi u/2)}{\sin \pi s/2}\right) \tag{18}$$

where

$$\begin{aligned} \alpha &= EE'(1 + vw\cos\theta) - nR^2 \\ \beta &= EE'(1 - vw\cos\theta) - nR^2 \\ \gamma &= 2(E^2 - R^2). \end{aligned} \tag{19}$$

Note that $\alpha + \beta = \gamma$ and as a result the exponential terms cancelled automatically. It also follows that A_4 reduces in the limit $R \to \infty$ to

$$A_4 \sim \left(\frac{\alpha}{\gamma}\right)^{2\alpha} \left(\frac{\beta}{\gamma}\right)^{2\beta} \left(\frac{\sin(-\pi t/2)\sin(-\pi u/2)}{\sin \pi s/2}\right). \tag{20}$$

It is easy to show that in the limit $R \to \infty$, $\alpha, \beta, \gamma \to \infty$. A tedious but straightforward computation using (12) and (13) shows that $|\alpha/\gamma| \leq 1$ and $|\beta/\gamma| \leq 1$ with either equality (but not both) being satisfied only for $n = 1$ and $\theta = 0, \pi$. In other words for $n \neq 1$, $A_4 \to 0$ exponentially as $R \to \infty$ for *all* scattering angles. So the only possible final states are those with $n_3 = n_4 = 1$ and $\theta = 0, \pi$. Note that for $n \neq 1$, there are no poles in A_4 at $\theta = 0, \pi$. Hence the 4-point function vanishes exponentially with the winding radius away from the poles, which exist only for $n = 1$. As mentioned above, we can repeat the calculation for $m' \neq m$, but we still get the same essential exponential decay in the winding radius, with the number of possible excited transitions bounded by a polynomial in R. In this limit, the amplitude is nonvanishing only for $m' = m$, $n = 1$ and $\theta = 0, \pi$.

A similar situation can also be shown to arise in the case of oppositely oriented strings. Going back to (5) and (6), if we set $n_1 = -n_2 = 1$ and $n_3 = -n_4 = n$, then a similar calculation to the one above shows that $A_4 \to 0$ exponentially as $R \to \infty$ except when $n = 1$ and $\theta = 0$ (or $n = -1$ and $\theta = \pi$). Again, the final states must be identical to the initial ones with zero scattering angle. In particular, there is no annihilation in the large radius limit. This would suggest that in a collision of oppositely oriented Dabholkar string solitons, the strings would collide under the influence of the attractive force between them but would emerge in the same final states. One can go further and show that for $n_1 = -n_2 = n$ and $n_3 = -n_4 = n'$, the amplitude vanishes in the large R limit except when $n = n'$ and $\theta = 0$ or $n = -n'$ and $\theta = \pi$. The calculation in this case is even more laborious, but is equally straightforward. Finally, one can generalize to the case of arbitrary incoming winding states. Once more, the amplitude is nonvanishing only when the final winding states are identical to the initial ones and the scattering angle is zero. In fact, the general case follows from the previous case by noting that since the kinematics of the left and right movers decouple and we can essentially consider one sector alone, the dynamics should not be affected by boosting, say, the right movers to a frame in which we have opposite winding, whence the trivial scattering result follows. In any event, the general case can be explicitly shown, and, although it is considerably more tedious than the simplest case, follows essentially the same line of argument.

The above calculations can be repeated for any other type of string, including the heterotic string[8]. The kinematics differ slightly from the tachyonic case but the 4-point functions are still dominated by an exponentially vanishing factor in the large radius limit, and are nonvanishing only at $\theta = 0, \pi$ and when the final states are identical to the initial states.

The above anaylsis represented a tree-level computation in string theory. It would be interesting to see whether the full quantum string loop scattering amplitudes still yield trivial scattering for the macroscopic winding states. In addition, it would be interesting to construct the Dabholkar analogs for the higher winding states as well as their full quantum string loop extensions.

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