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Can Q-balls save the Universe from Intermediate-Scale Phase Transitions?

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Abstract

We discuss the possible role of non-topological solitons (Q-balls) during first-order phase transitions in the early Universe. We argue that Q-balls cannot mediate such a transition unless the self-coupling in the effective potential is large, in which case the tunnelling probability is in any case high. A corollary of our analysis is that Q-balls do not solve the entropy problems of flat potentials such as those of Coleman and Weinberg, no-scale models and superstring models with intermediate scales. We also comment on other problems of superstring models with intermediate scales.

A recurrent cosmological problem for particle physics models is the generation of excess entropy at a phase transition. This arises in models with a first-order phase transition, and is particularly serious when the effective potential is very flat and the scalar fields self-interact weakly. Examples of such flat potentials include Coleman-Weinberg[1] models in gauge theories, and no-scale supersymmetric models[2]. Examples of the latter are effective supergravity or superstring models with a large symmetry-breaking scale m_I intermediate between m_W and m_P[3] (the electroweak and Planck mass scales respectively). A large intermediate scale $m_I \gg m_W$ is assumed to arise because of the absence of renormalizable self-interactions along the direction of symmetry breaking. Such flatness is required because the supersymmetry-breaking scalar masses must be O(1 TeV). The flatness is broken only by those masses, which have scale-dependent radiative corrections, and possibly by non-renormalizable terms in the effective potential. In such intermediate-scale models, the phase transitions only occurs after the temperature falls to $T \sim O(m_W)$, and results in excess entropy production unless[4,5] $m_I \leq 10^{12}$ GeV.*

The problem of excess entropy release during the phase transition appears because the probability of tunnelling from the false vacuum to the true vacuum is negligible when $T \gg m_W$. The question therefore arises whether some other mechanism could complete the transition more rapidly and avoid the excess entropy production. It is known that under certain conditions the spectrum of a scalar field theory with a global U(1) symmetry

*As we comment later, this is only one of many problems with intermediate scales in superstring models.

contains classically stable non-topological soliton states called Q-balls[6]. It has been suggested that these could play an important role in mediating a symmetry-breaking phase transition, and the rate of cosmological production of non-topological solitons has been discussed[7]. However, the role Q-balls might play[6] in mediating a transition has not been worked out in detail, and the possibility that they might avoid the entropy problems of intermediate-scale models has not been addressed.

In this paper we explore general necessary conditions for Q-balls to be able to mediate a cosmological phase transition, taking into account the fact that they have a minimum possible charge Q_{min} determined by surface effects, and a maximum possible charge Q_{max}[8] if the global symmetry is gauged as in most models of interest. We find that Q-balls can be important only if the scalar self-coupling λ is large, which is not the case for Coleman-Weinberg[1] and intermediate-scale models[2,3]. Indeed if λ is large enough for Q-balls to be important, the tunnelling amplitude is also large, so that the entropy problem does not arise. We conclude by reminding[5,9] the reader that the intermediate-scale models[3,10] whose entropy problem is not solved by Q-balls also have many other phenomenological difficulties.

We consider[6] an effective field theory with a global U(1) symmetry, given by the lagrangian

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - U(\Phi) \tag{1}$$

where $\Phi = \sqrt{\phi^* \phi}$. If

$$\omega_o^2 = \min(2U/\Phi^2) = 2U_o/\Phi_o^2 < \mu^2 = U''(0) = 2U/\Phi^2\big|_{\Phi = 0} \tag{2}$$

then the model has a stable non-topological soliton, the Q-ball, with $\Phi = \Phi(r)e^{-i\omega t}$ where $\Phi(r)$ is approximately a step function $\Phi(r) \simeq \Phi_o \theta(R-r)$ and R is the radius of the soliton. The conserved charge is $Q = 2\omega \int d^3x |\Phi(r)|^2$, where $\omega \to \omega_o$ as $|\phi| \to \infty$. The energy of the Q-ball can be written in the form

$$E = A/R^3 + BR^3 + CR^2 \tag{3}$$

where the constants are given by[6)]

$$A/R^3 = 3Q^2/8\pi \int \Phi^2 d^3x,\ BR^3 = \frac{4\pi}{3} \int d^3x\ U(\Phi),\ C = 4\pi \int d\phi \sqrt{2\hat{U}} \tag{4}$$

and $\hat{U} = U - \frac{1}{2}\omega_o^2 \Phi^2$. The values of these constants for the lowest energy configurations are determined by minimizing Eq. (3) with respect to R, yielding the stable Q-ball solution. A generic potential $U(\Phi)$ which would have a stable Q-ball is shown in the figure. We parametrize it as

$$U = \lambda^2\ f(\Phi/\Phi_A)\Phi_A^4 \tag{5}$$

where λ is an overall scale which may be very small, Φ_A is the location of the asymmetric minimum of $U(\Phi)$, and we assume there are no other very small (or large) parameters in the reduced function $f(x \equiv \Phi/\Phi_A)$. The scalar potential $U(\Phi)$ is temperature-dependent, and we will be using the generic form (5) slightly above the critical temperature T_c at which $U(0) \equiv U_S = U(\Phi_A) \equiv U_A$. As the temperature falls, U_A falls below U_S and the asymmetric minimum becomes energetically favoured. Since the Q-ball interpolates between $\Phi = 0$ and $\Phi = \Phi_o$ which approaches Φ_A as $U_A \to U_S$, our question is whether the Q-ball could mediate the phase transition $\Phi = 0 \rightarrow \Phi_A$, which one would normally expect to be first order and to generate large entropy if $\lambda \ll 1$ as in the models[1,2,3] which motivate our interest.

We must first determine the probability of Q-ball formation in the cooling Universe. We recall that in normal thermal equilibrium the relative probability of finding a given value of Φ is $P \sim \exp(-F/T)$ where F is the free energy. However, when the transition rate between two vacua falls below the expansion rate of the Universe, which occurs at the Ginzburg temperature T_G[11], the relative probabilities freeze. We learn from ref. 7 that then

$$P_A/P_S \simeq \exp(-U_A/U_M) \qquad (6)$$

where U_M is the local maximum of the potential indicated in the figure. If $P_A > 0.31$, the formation of the symmetric vacuum state is so commonplace that there are infinite domains where $\Phi = \Phi_A$, according to percolation studies[12]. This would occur if $U_A \leq 0.8U_M$ at the Ginzburg temperature. Even in the limiting case where $U_A/U_M \rightarrow 1^-$, equation (6) tells us that $P_A = 0.26$, suggesting that the formation of a Q-ball could be a likely event, even though each one would be surrounded by an infinite volume of symmetric vacuum.

Once a bubble of the asymmetric vacuum, i.e., a Q-ball, has formed, it tends to grow without limit as the temperature falls to T_c. To see this, neglect the last (surface) term in equation (3), in which case the energy for fixed Q is minimized by a configuration with volume[6]

$$V = \frac{4\pi R^3}{3} = Q/\sqrt{2}\Phi_A^2 U_A \rightarrow \infty \tag{7}$$

as $T \rightarrow T_c$ and $U_A \rightarrow 0$. Thus one might hope that even the bubbles generated by the minimum probability $P_A = 0.26$ for the asymmetric vacuum would expand to take over the Universe as $T \rightarrow T_c$.

However, this first guess ignores the question whether the corresponding Q-balls have charges within the allowed range. When a domain of asymmetric vacuum forms at T_G, its characteristic size is given by the correlation volume $V_\xi = (2\xi)^3$, where ξ is the correlation length. We estimate V_ξ by identifying the transition rate $\Gamma_T \simeq T \exp(-U_M V_\xi/T)$ with the Hubble expansion rate H to obtain

$$V_\xi \simeq T_G L/U_M: \quad L = \ell n(T_G/H) \tag{8}$$

In cases of interest where $H \sim T_G^2/m_P$ and $m_P >> T_G \gtrsim 1$ MeV we expect $L \lesssim 50$. Since the number density $n \sim T^3$ for each particle species, we expect $N \sim T_G^3 V_\xi$ charged particles in each correlation volume, and hence a net charge

$$Q \simeq N^{1/2} \simeq T_G^{3/2} V_\xi^{1/2} \simeq V_\xi^2 \, U_M^{3/2}/L^{3/2} \tag{9}$$

in each bubble of new asymmetric vacuum. The desired Q-ball will indeed be stable if $Q_{min} < Q < Q_{max}$, where we now specify these limits on the Q-ball charge.

A lower limit Q_{min} comes from taking account of the last (surface) term in the expression (3) for the energy for the Q-ball[7]. E is minimized by

$$E_{min} \simeq 2BQR_o^3 + CQ^{2/3}R_o^2 \tag{10}$$

where

$$R_o^2 = (A/BQ^2)^{1/3} \tag{11}$$

and $2BR_o^3 = \omega_o$ would have been the energy per unit charge in the absence of any surface energy. The condition for a Q-ball to exist is

$$E_{min} < Q\ \mu \tag{12}$$

which requires

$$Q > Q_{min} = (\frac{CR_o^2}{\mu - \omega_o})^3 \tag{13}$$

We note that this result was obtained by assuming as a first approximation that the surface energy was much smaller than the volume energy. If this were not so, Q-balls probably would not exist. The surface energy would start to dominate when $CR^2 \gtrsim BR^3$ or $Q \gtrsim 2C^2/B\omega_o = Q_*$. It is clear that $Q_* > Q_{min}$ if $\mu > 3/2\ \omega_o$, in which case the following analysis would need to be modified somewhat. However, it is reasonable to assume that ω_o is not much smaller than μ at the Ginzburg temperature when the Q-ball would be formed. For a potential of the generic form (5), we have

$$C = O(1) \times 4\pi\lambda\ \Phi_A^3, \qquad \mu - \omega_o = O(1) \times \lambda\Phi_A \tag{14}$$

so that

$$Q_{min} = 0(1) \times \frac{18\pi}{\lambda^2} \tag{15}$$

which we will shortly compare with the estimate of Q in equation (9).

However, before doing so we recall that there is also an upper limit Q_{max} on the charge which holds if the U(1) symmetry is gauged[8]. In this case we must impose

$$\omega_o \lesssim \mu - \frac{e^2}{4\pi}\frac{Q}{R} \tag{16}$$

which has the physical interpretation that it is energetically disfavored to bring a charged particle in from infinity and add it on the surface of the Q-ball ($e = \sqrt{4\pi\alpha}$ is the U(1) gauge coupling). Equality in equation (16) gives

$$Q_{max} = \left(\frac{\mu - \omega_o}{\alpha}\right)^{3/2} \left(\frac{3}{4\pi\sqrt{2\phi_A^2 U_a}}\right)^{1/2} \tag{17}$$

where we have used equation (7), as is appropriate for large Q-balls. For the generic potential (5), the upper bound (17) can be written as

$$Q_{max} = 0(1) \times \frac{\lambda}{\alpha^{3/2}} \sqrt{\frac{3}{4\pi\sqrt{2}}} \tag{18}$$

which we will also compare with the estimate of Q in equation (9).

The correlation length ξ appearing in (9) is simply given by $\xi = 1/m(T_G)$, where m(T) is the effective temperature-dependent mass:

$$m^2(T) = m_o^2 + 0(\lambda^2)T^2 + 0(g^2)T^2 \tag{19}$$

where g is a generic gauge coupling constant that need not necessarily be identified with the U(1) coupling e. Since our motivation is the case where $\lambda \ll 1$, we assume that the last term in equation (19) dominates, in which case

$$V_\xi \simeq \xi^3 = (m(T_G))^{-3} \simeq g^{-3}T_G^{-3} \tag{20}$$

Substituting this into equation (8), we find

$$T_G \simeq 0(1) \times \frac{\lambda^{1/2}}{L^{1/4}g^{3/4}} \tag{21}$$

and

$$V_\xi \simeq 0(1) \times \frac{1}{\lambda^{3/2}} \frac{1}{g^{3/4}} L^{3/4} \tag{22}$$

so that from equation (9)

$$Q = 0(1) \times \frac{1}{g^{3/2}} \tag{23}$$

The double condition $Q_{min} < Q < Q_{max}$ therefore becomes

$$0(1) \times \frac{18\pi}{\lambda^2} < \frac{0(1)}{g^{3/2}} < 0(1) \frac{\lambda}{\alpha^{3/2}} \sqrt{\frac{3}{4\pi\sqrt{2}}} \tag{24}$$

The first inequality cannot be evaded by deciding not gauge the U(1): it imposes

$$\lambda^2 > 0(1) \times 18\pi\ g^{3/2} \qquad (25)$$

which conflicts with the expectation that $\lambda \ll 1$, and is not satisfied by any of the flat potentials of interest to us[1,2,3,10]. If we were to suppose that the ϕ field has no gauge interactions at all, we would have $\xi \sim 1/\lambda\ T_G$ and hence $T_G \simeq 0(1) \times \frac{1}{L^{1/4}\lambda^{1/4}}$, $V_\xi \simeq 0(1) \times L^{3/4}/\lambda^{9/4}$, and $Q \simeq 0(1)/\lambda^{3/2}$. Even in this case the $Q_{min} < Q$ condition would impose

$$\lambda^{1/2} > 0(1) \times (18\pi) \qquad (25')$$

which is not very small! The requirement $Q < Q_{max}$, which need be imposed only if the U(1) is gauged, entails

$$\lambda > 0(1) \times \sqrt{\frac{4\pi\sqrt{2}}{3}}\ \frac{\alpha^{3/2}}{g^{3/2}} \qquad (26)$$

if the ϕ field has other gauge interactions, which becomes

$$\lambda > \frac{0(1)\sqrt{\sqrt{2}}\ e^{3/2}}{4\pi\sqrt{3}} \qquad (26')$$

if it has no other gauge interactions.

We conclude from the inequality (25) [or (25′)] that Q-balls would be copiously produced and hence able to facilitate the phase transition only if

λ is large, in which case the phase transition could in any case proceed via other mechanisms, such as quantum tunnelling, without generating excess entropy. If λ is O(1), the gauged U(1) requirement (26) [or (26')] is likely to be satisfied. We have seen that Q-balls are not useful for alleviating the cosmological problems of theories with flat potentials, such as Coleman-Weinberg[1)], no-scale[2)], or intermediate-scale models[3,10)].

For completeness, we also remind the reader that intermediate-scale models[3,10)] have many other problems too. As we have already emphasized, the fact that a large intermediate scale is generated along an almost flat direction of the effective potential brings the danger of excessive entropy generation when the Universe makes the transition to the asymmetric phase[4,5)]. The only clear solution we know to this problem is that of ref. 13, where strong interaction effects at a scale $\Lambda \sim 10^{11}$ GeV destabilize the symmetric vacuum. In the context of superstring models, this solution can operate[14)] in the flipped SU(5) x U(1) model of ref. 15. Models with smaller gauge group factors in the observable sector, such as $[SU(3)]^{3}$[10)] or SU(4) x SU(2)x SU(2)[16)], cannot exploit this solution and therefore suffer a priori from a disastrous entropy problem.

They have other difficulties as well. To be compatible with proton stability limits, they need $m_I > 10^{16}$ GeV[9)], and renormalization group analyses[17,10)] suggest that $m_I > 10^{17}$ GeV is needed to keep $\sin^2\theta_W < 0.235$ as required by experiments. This requires the absence of non-renormalizable terms $\propto (\phi^*\phi)^n$ with $n < 7$ in the flat direction of the potential. On the other hand, other non-renormalizable terms with lower powers n are needed to give acceptably large masses to many unseen mirror particles[10)]. In the absence of any systematic procedure for calculating such non-renormalizable terms, it must just be assumed ad hoc that the undesirable terms are absent and the desirable ones are present. If this is done in the model of ref. 3, 10, so that $m_I/m_P \sim 10^{-1}$, the renormalization group analysis[10] then indicates that the gauge couplings in the different SU(3) factors differ by factors ~ 2 at m_I, which conflicts with perturbative string expectations[18].

It has also been argued[19,10] that the presence of a matter parity in the theory can guarantee the absences of proton decay and of new particle states which could lead to severe problems with flavor-changing neutral currents. A judicious neglect of "accidental" zeros in the quark and lepton mass matrices was a key assumption in the analysis of ref. 10. However, other discrete symmetries necessarily present in $CP^3 \times CP^3$ Calabi-Yau models automatically create a plethora of accidental zeros. Thus one is left with a resultant low energy spectrum[9] with extra light particles and/or light states which are linear combinations of standard and exotic fields, the latter being an expected problem in any rank-6 superstring model[20]. Other difficulties including the protection of the Higgs mass were addressed in ref. 21.

If m_I/m_P is indeed $O(10^{-1})$, it is no longer sufficient to use the leading logarithimic approximation[22] to the renormalization group equations to investigate whether and when gauge symmetry breaking occurs, as was done in ref. 23. Instead, one should use the full one-loop effective potential as was done in ref. 24. In the leading logarithimic approximation, renormalization group evolution led to a negative mass^2 for one or more of the standard model singlets thus triggering spontaneous symmetry breaking, but typically only when substantial bare scalar masses are present[23]. When gaugino masses are the dominant source of supersymmetry breaking, symmetry breaking in general will not occur[5,9]. We note, moreover, that most authors[25] favor the gaugino mass $m_{1/2}$ as the dominant source of supersymmetry breaking in the observable sector, whereas in ref. 21 it was mostly taken to be the scalar mass m_o. When the full one-loop effective potential is used, despite the fact that scalar masses still do not become negative at the origin, the phase transition proceeds[24] via strong coupling phenomena as was advocated in refs. 13 and 14.

In addition to these technical objections, we also feel that ref. 3, and 10 do not resolve the problem of ambiguity in the choice of the intermediate-scale symmetry breaking direction. It is generally assumed[10] that the fields developing large v.e.v.'s respect matter parity[19] which is necessary but not sufficient for approximate proton stability, the absence of $\Delta L \neq 0$ interactions, flavor-changing neutral currents, etc.[9], but this has not been proven. Indeed, the problem of ambiguity already appears at the compactification scale, since it is normally assumed without proof that specific values of the Calabi-Yau manifold moduli are chosen so as to obtain certain essential discrete symmetries[3,10].

We conclude that intermediate-scale models still have a host of problems, and regret the fact that Q-balls do not solve even one of them.

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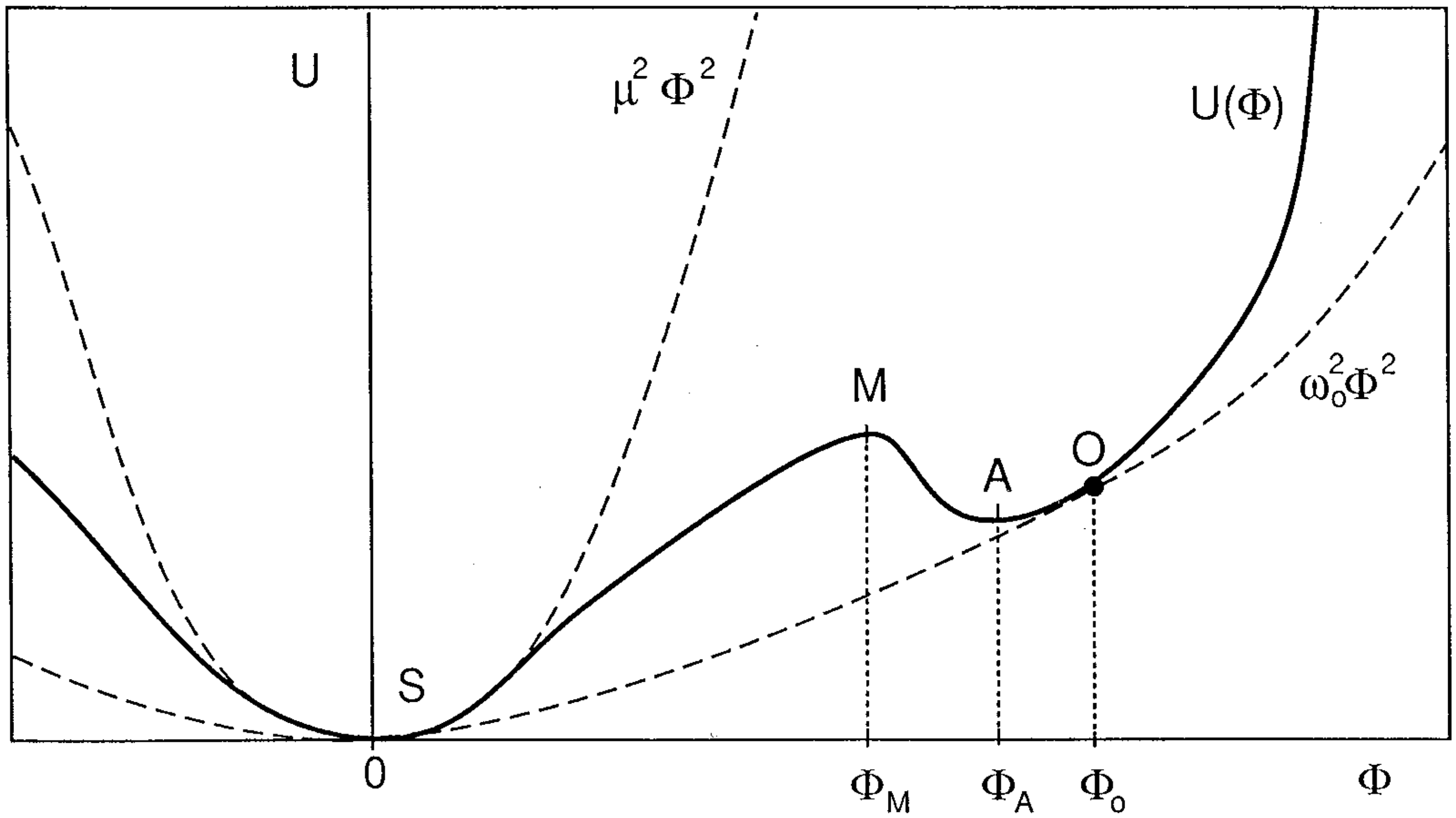
Figure Caption

The effective potential $U(\Phi)$ is minimized at the point S: $\Phi = 0$, near which $U \simeq \mu^2\Phi^2$. There is a local maximum at the point M: $\Phi = \Phi_M$ and a local minimum at the point A: $\Phi = \Phi_A$. The quantity $2U/\Phi^2$ is minimized at the point O: $\Phi = \Phi_o$.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