Inter-Jurisdiction Migration and the Fiscal Policies of Local Governments

(January 18, 2019)

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Abstract

This paper first analyzes a fiscal-policy game between two jurisdictions connected by mutual migration and obtains two main results. (i) As the mutual migration intensifies, both jurisdictions in the Nash equilibrium choose more public consumption, less public investment, and more total spending that is entirely financed by debt. (ii) The first-best allocation can be achieved through Nash play by imposing the restriction that public consumption should be financed by a contemporary tax and not by borrowing. The paper then goes on to analyze a model with one-directional migration and obtains results on how migration affects the fiscal policies of both the jurisdiction of migration destination and the jurisdiction of migration origin.

Keywords: local government debt; migration; fiscal externalities; debt limits
JEL Classification: H23; H74

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1. Introduction

The relationship between migrations and the fiscal policies of local governments has drawn extensive economic analyses.[1] It is properly emphasized in these studies that the causal effect in this relationship could run in both directions. On the one hand, the high tax/debt policy in a jurisdiction may cause its residents to migrate to another jurisdiction, a mechanism referred to as tax/debt-driven migration. On the other hand, the presence of migration, or even the threat to migrate, helps shape the fiscal policies in jurisdictions that are connected through migration.

Most existing studies on the effects of migration on the fiscal policies of local governments focus on the tax/debt-driven migration that is endogenously determined in a model of optimal fiscal policy. As a result of such endogeneity, the analysis tends to be quite complicated and the results obtained are often not clear cut. In reality, however, people may change residence locations for reasons other than the tax/debt consideration. For example, people may move to a new location for marriage, a better job opportunity, a hobby or schooling.

In this paper we analyze the effects of exogenous, non-tax/debt-driven migration on the fiscal policies of local governments. Because of the possibility of migrating to another jurisdiction later in life, and as a result, not being responsible for the debt repayment in one's original jurisdiction, the residents in a jurisdiction have a tendency to run excessive debt. In this aspect, our analysis is closely related to those that have previously examined the debt-increasing effect of migration, including Daly (1969), Oates (1972), Bruce (1995) and Schultz and Sjostrom (2001), among others. However, our analysis goes beyond those earlier analyses by making a distinction between government spending on public consumption (which produces a publicly-

[1] See, for example, Mirrlees (1982), Wilson (1982), Wildasin (1994), Leite-Monteiro (1997), Lehmann et al. (2014) and Dai (2017).

provided good in period 1) and government spending on public investment (which produces a publicly-provided good in period 2). With a positive probability of out-migrating in the future, residents in a jurisdiction view public investment in their jurisdiction as less valuable than when migration is impossible. Therefore, spending on public consumption and spending on public investment should be treated as separate variables in the model of the fiscal policies of local governments that are connected by migration.[2]

Making a distinction between these two types of government spending facilitates obtaining two new, clear-cut theoretical findings within a two-jurisdiction, two-period model in which initially identical residents in each jurisdiction choose both public consumption and public investment in period 1, before knowing their migration status in period 2. First, as the between-jurisdiction migration intensifies, both jurisdictions in the Nash equilibrium incur more spending on public consumption, less spending on public investment, and more total spending. Further, this total spending will equal the level of government debt, because in the Nash equilibrium both jurisdictions opt for debt financing in the presence of migration. Compared to the earlier studies on the effect of migration on local governments' debt levels that do not treat public consumption and public investment separately, the result above explicitly attributes the increased debt level due to migration to increased public consumption (or more precisely, government spending incurred to provide goods in the current period). Indeed, in response to a heightened probability of migration, public investment (or more precisely, government spending incurred in the present to provide goods in the future period) actually decreases exactly when the total government spending increases.

[2] The focus of our analysis is therefore on the "horizontal fiscal externalities" through inter-jurisdiction migration, compared to analyses of "vertical fiscal externalities" that have a focus on the fiscal-policy interactions between a national government and a sub-national government (Besley and Rosen 1998, Dahlby and Wilson 2003, and Aronsson 2010).

Second, the first-best allocation can be achieved through Nash play by imposing the restriction that public consumption should be financed only by a contemporary tax and not by government borrowing.[3] This result provides a theoretical justification, based on migration, for the wide-spread practice of national governments imposing debt limits on local governments.[4] In contrast, a balanced-budget restriction, which prohibits government borrowing, would not achieve the first-best allocation. This result is the same in spirit as that obtained by Bassetto and Sargent (2006) in a model of overlapping generations without migration. They find that whenever demographics do not imply Ricardian equivalence, a "golden rule" that separates capital and ordinary account budgets and finances only capital spending with debt is approximately optimal.

In the real world, and especially when the jurisdictions are countries rather than subnational regions within a country, migration is often one-directional, where one of the two linked jurisdictions is the migration destination (e.g., the U.S., Australia or Canada) and the other is the migration origin (e.g., Puerto Rico, Mexico, China or India). So the paper goes on to analyze how the intensity of one-directional migration affects the fiscal policies of the two jurisdictions. It finds that migration's effects on the migration destination are very similar to those obtained in the model with mutual migration, and that migration's effects on the migration origin critically depend on how responsive the fiscal policies of the migration destination are to migration.

The main results are established in Section 2 in a model with two identical jurisdictions, separable utility functions, a perfect dichotomy between public consumption and public

[3] While this restriction says nothing about how the spending on public investment should be financed, both jurisdictions will opt for financing public investment exclusively with debt in the Nash equilibrium.

[4] For discussions of debt limits imposed by various national/federal governments, see Bird and Slack (1983), Aronson and Hilley (1986) and Mathews (1986).

investment, and no cross-jurisdiction debt holding. The focus of this section is on the effects of mutual migration on two linked jurisdictions' fiscal policies. Section 3 extends the analysis to the asymmetric case where the two linked jurisdictions/countries can be of different sizes and migration is one-directional. The analysis in this section addresses migration's effects on two linked jurisdictions/countries' fiscal policies in situations where one jurisdiction/country is the destination of migration and the other is the origin of migration. Section 4 shows that the results obtained in Section 2 can be generalized to the case of non-separable utility functions, the case where the dichotomy between public consumption and public investment is imperfect (i.e. the case where the publicly-provided goods are durable), and the case of cross-jurisdiction debt holding. Section 5 concludes.

2. A Fiscal-Policy Game between Two Identical Jurisdictions with Mutual Migration

We construct a fiscal-policy game between two identical jurisdictions, denoted A and B. These jurisdictions have the same number of residents, among other things. Individuals start off identical within and between jurisdictions, except that they are located in different residence locations.

Suppose that each individual lives for two periods with income y_1 in period 1 and y_2 in period 2, and each individual has a lifetime utility function

$$u_1(c_1) + g_1(G_1) + u_2(c_2) + g_2(G_2)\,, \tag{1}$$

where c_1 and c_2 are respectively the private consumption in periods 1 and 2, and G_1 and G_2 are respectively the publicly-provided goods in periods 1 and 2, where G_1 and G_2 are per-capita amounts. Note that the publicly-provided goods in this paper are "private" or "rival" goods, rather than "public" or "non-rival" goods. While in general publicly-provided goods can be either "private" or "public" goods, we focus on publicly-provided "private" goods because the present paper is mainly concerned with government financing in the presence of migration, not with the optimal provision of non-rival "public" goods.[5]

All four functions in (1) are assumed to be strictly increasing and strictly concave. The lifetime utility function of (1) is both timely separable and separable in the private consumption and the publicly-provided good in the same period. For the issues analyzed in the present paper, this "double separability" assumption is not overly restrictive, but it greatly simplifies the technical aspect of the analysis.[6]

We focus on the decisions of the representative individual in jurisdiction A, because those of the representative individual in jurisdiction B are exactly symmetric. In period 1, the representative individual in jurisdiction A decides on the amount invested in productive capital (i.e., private saving) k^A ($k^A < 0$ would indicate borrowing instead of holding capital), public consumption G_1^A and public investment G_2^A, all the amounts being stated in per-capita terms. Note that variables G_1^A and G_2^A serve a dual purpose. On the output side, G_1^A and G_2^A are publicly-provided goods in period 1 and period 2, respectively. One implicit assumption – that G_1^A and G_2^A are non-durable goods – is made here for a sharp interpretation of the results.[7] On the input side, G_1^A and G_2^A are both government spending in period 1, and the important difference is that G_1^A is used to provide publicly available goods and services in period 1

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[5] Nevertheless, the main results of the paper obtained in Sections 2 and 4 still hold when G_1 and G_2 are publicly-provided "public" goods (with appropriate interpretation of functions g_1 and g_2) because the number of residents in each jurisdiction stays the same.

[6] In Section 4, we consider a more general lifetime utility function $u(c_1, G_1) + v(c_2, G_2)$, where the standard timely separability is maintained, but the private consumption and the publicly-provided good in the same period are allowed to be non-separable.

[7] In Section 4, we consider a more general situation where G_1^A is durable with various degrees.

whereas G_2^A is used to provide publicly available goods and services in period 2. Based on the input side interpretation, G_1^A is referred to as spending on public consumption (or simply public consumption), and G_2^A as spending on public investment (or simply public investment). Examples of G_1^A include spending on transfer payments, health care and education,[8] whereas examples of G_2^A include spending on infrastructure, R&D and environment.

The government can finance its total spending in period 1, $G_1^A + G_2^A$, by a mix of tax collections or debt finance. Suppose that α_A is the portion of the total spending that is financed by debt. To focus on the main forces at work for the results derived in this model, we initially assume that the debt issued by a jurisdiction is exclusively held by the residents of that jurisdiction.[9] Then the representative individual holds bonds in the amount of $\alpha_A\left(G_1^A+G_2^A\right)$, which will be repaid to him with interest in period 2. Regardless of the specific mix of tax and debt, however, the representative individual's private consumption in period 1 is

$$c_1^A = y_1 - k^A - G_1^A - G_2^A,$$

and this will be true with or without a positive probability of migration.

At the beginning of period 2, a fixed portion, denoted p, of the population in jurisdiction A will be randomly selected to migrate to jurisdiction B. At the same time, the same fixed portion of the population in jurisdiction B migrate to jurisdiction A, so that the number of individuals in each jurisdiction will not change after the process of mutual migration. We

[8] Even though spending on education is usually treated as investment, it is "public consumption" according to the terminology used in this paper because the (capitalized) benefits of education accrue to the individuals who receive the education.

[9] In Section 4, we relax this assumption to allow some share of one jurisdiction's debt to be held by residents of the other jurisdiction. It is shown that the symmetric Nash equilibrium is characterized by the same set of conditions regardless of how much cross-jurisdiction debt holding is allowed.

assume that the migration decision is exogenous – that is, people migrate between jurisdictions for reasons that are unrelated to the fiscal policy issues considered here (e.g., schooling or marriage) – to focus on the externalities, through migration, of one jurisdiction's fiscal policy on another's.

If staying put in period 2 (with probability 1-p), the representative individual will pay for the debt repayment in jurisdiction A, and also receive utility from the public investment in jurisdiction A. In particular, his private consumption is

$$c_2^{A,S} = y_2 + k^A(1+r) + \alpha_A(G_1^A + G_2^A)(1+r) - \alpha_A(G_1^A + G_2^A)(1+r) = y_2 + k^A(1+r),$$

where r is the rate of return earned on productive capital as well as government bonds, and the debt repayment the government owes him and the tax required to make the payment cancel out. This is why in the absence of migration an individual is indifferent between tax or bond finance of period 1 government spending.

If migrating from jurisdiction A to B in period 2 (with probability p), the representative individual will be subject to the government in jurisdiction B, paying taxes in period 2 for the debt incurred by jurisdiction B, and receiving benefit from public investment that was undertaken in jurisdiction B. In particular, his private consumption is

$$c_2^{A,M} = y_2 + k^A(1+r) + \alpha_A(G_1^A + G_2^A)(1+r) - \alpha_B(G_1^B + G_2^B)(1+r),$$

where G_1^B and G_2^B are the per-capita public consumption and per-capita public investment in jurisdiction B, respectively.

Note that regardless of what jurisdiction B chooses (including the value of α_B), and regardless of the values of k^A, G_1^A and G_2^A, the representative individual's $c_2^{A,M}$ is maximized when $\alpha_A = 1$, whereas c_1^A and $c_2^{A,S}$ are unaffected by the choice of α_A . Therefore, in period 1 the representative individual in jurisdiction A will always choose to finance the entire total spending in period 1 with debt, that is $\alpha_A = 1$ is part of jurisdiction A's optimal fiscal policy. Intuitively, with a positive probability of migrating to jurisdiction B and not being responsible for debt retirement in jurisdiction A, the representative individual in jurisdiction A views debt financing as less costly than contemporary-tax financing. The same argument leads to $\alpha_B = 1$.

Therefore, the representative individual in jurisdiction A chooses k^A, G_1^A and G_2^A to maximize his expected lifetime utility

$$\begin{aligned} &u_1\left(y_1 - k^A - G_1^A - G_2^A\right) + g_1(G_1^A) + (1-p)u_2\left(y_2 + k^A(1+r)\right) + (1-p)g_2(G_2^A) \\ &+ pu_2\left(y_2 + k^A(1+r) + (G_1^A + G_2^A)(1+r) - (G_1^B + G_2^B)(1+r)\right) + pg_2(G_2^B), \end{aligned} \tag{2}$$

taking G_1^B and G_2^B as given. The first-order conditions with respect to k^A, G_1^A and G_2^A are respectively

$$\begin{aligned} &-u_1'\left(y_1 - k^A - G_1^A - G_2^A\right) + (1+r)(1-p)u_2'\left(y_2 + k^A(1+r)\right) \\ &\qquad + (1+r)pu_2'\left(y_2 + k^A(1+r) + (G_1^A + G_2^A)(1+r) - (G_1^B + G_2^B)(1+r)\right) = 0 \\ &-u_1'\left(y_1 - k^A - G_1^A - G_2^A\right) + g_1'(G_1^A) + (1+r)pu_2'\left(y_2 + k^A(1+r) + (G_1^A + G_2^A)(1+r) - (G_1^B + G_2^B)(1+r)\right) = 0 \\ &-u_1'\left(y_1 - k^A - G_1^A - G_2^A\right) + (1-p)g_2'(G_2^A) + (1+r)pu_2'\left(y_2 + k^A(1+r) + (G_1^A + G_2^A)(1+r) - (G_1^B + G_2^B)(1+r)\right) = 0. \end{aligned} \tag{3}$$

It is straightforward to show that the corresponding Hessian matrix is negative definite, and therefore the second-order condition for the expected lifetime utility maximization problem is satisfied.

2. 1. The Effect of Migration on the Symmetric Nash Equilibrium

Imposing the symmetry conditions, that $k^A = k^B = k^*$, $G_1^A = G_1^B = G_1^*$ and $G_2^A = G_2^B = G_2^*$, and rearranging the three equations in (3), the symmetric Nash equilibrium (k^*, G_1^*, G_2^*) is determined by the following equations:

$$-u_1'\left(y_1-k^*-G_1^*-G_2^*\right)+(1+r)u_2'\left(y_2+k^*(1+r)\right)=0$$
$$-(1-p)u_1'\left(y_1-k^*-G_1^*-G_2^*\right)+g_1'(G_1^*)=0 \tag{4}$$
$$-u_1'\left(y_1-k^*-G_1^*-G_2^*\right)+g_2'(G_2^*)=0.$$

Proposition 1. The comparative statics of the symmetric Nash equilibrium (k^*, G_1^*, G_2^*) with respect to p are $\frac{dk^*}{dp}<0, \frac{dG_1^*}{dp}>0, \frac{dG_2^*}{dp}<0$ and $\frac{d(G_1^*+G_2^*)}{dp}>0$.

Proof: See the appendix.

Proposition 1 says that as between-jurisdiction migration (p) increases, residents in both jurisdictions hold less productive capital ($\frac{dk^*}{dp}<0$), both jurisdictions provide more public consumption ($\frac{dG_1^*}{dp}>0$) and less public investment ($\frac{dG_2^*}{dp}<0$), and both jurisdictions incur more debt to finance a larger total spending in period 1 ($\frac{d(G_1^*+G_2^*)}{dp}>0$). These results are quite intuitive. With a larger chance to move to another jurisdiction later, the representative individual in each jurisdiction views public investment in his own jurisdiction as less valuable and debt financing less costly. Therefore, public investment in each jurisdiction becomes smaller ($\frac{dG_2^*}{dp}<0$), and both jurisdictions incur more debt ($\frac{d(G_1^*+G_2^*)}{dp}>0$). The combination of more total government spending and less public investment implies an increase in public consumption ($\frac{dG_1^*}{dp}>0$). As for people holding less productive capital ($\frac{dk^*}{dp}<0$), it is the implication of government bonds being a perfect substitute for productive capital and people holding more government bonds.

The key to the results in Proposition 1 is that an increase in outbound migration causes both the costs of debt-finance of current and prepaid future services and the benefits of future services to decrease. Therefore, migration's effects on a jurisdiction's fiscal policy obtained here are very similar to the effects of mortality or politicians' short-term bias on the fiscal policy of a jurisdiction without migration.

2.2. The First-Best Allocation and Optimal Financing of Government Spending

The first-best allocation in both jurisdictions, denoted $(\hat{k}, \hat{G}_1, \hat{G}_2)$, maximizes

$$u_1\left(y_1 - k - G_1 - G_2\right) + g_1(G_1) + u_2\left(y_2 + k(1+r)\right) + g_2(G_2). \tag{5}$$

$(\hat{k}, \hat{G}_1, \hat{G}_2)$ is determined by the following first-order conditions:

$$\begin{aligned} &-u_1'\left(y_1 - \hat{k} - \hat{G}_1 - \hat{G}_2\right) + (1+r)u_2'\left(y_2 + \hat{k}(1+r)\right) = 0 \\ &-u_1'\left(y_1 - \hat{k} - \hat{G}_1 - \hat{G}_2\right) + g_1'(\hat{G}_1) = 0 \\ &-u_1'\left(y_1 - \hat{k} - \hat{G}_1 - \hat{G}_2\right) + g_2'(\hat{G}_2) = 0. \end{aligned} \tag{6}$$

We can make two immediate observations regarding the relationship between the first-best allocation $(\hat{k}, \hat{G}_1, \hat{G}_2)$ and the Nash equilibrium allocation (k^*, G_1^*, G_2^*) in the last subsection. First, it is clear by comparing (4) and (6) that the two allocations are identical when p = 0. This result is easy to understand because, in our model, the only factor that causes the Nash equilibrium allocation to be suboptimal is the externalities of one jurisdiction's fiscal policy on another via migration. Second, individuals in both jurisdictions are strictly better off under the first-best allocation $(\hat{k}, \hat{G}_1, \hat{G}_2)$ than under the Nash equilibrium allocation (k^*, G_1^*, G_2^*) when p > 0. To see this, note that the expected lifetime utility of (2) is

$$u_1\left(y_1 - k^* - G_1^* - G_2^*\right) + g_1(G_1^*) + u_2\left(y_2 + k^*(1+r)\right) + g_2(G_2^*) \tag{7}$$

at the Nash equilibrium allocation (k^*, G_1^*, G_2^*), and (7) is simply the lifetime utility function (5) evaluated at (k^*, G_1^*, G_2^*), whereas (5) is maximized at the first-best allocation $(\hat{k}, \hat{G}_1, \hat{G}_2)$ by definition.

As pointed out earlier in this section, the Nash equilibrium has the entire government spending in period 1 (i.e., $G_1^A + G_2^A$ per-capita in jurisdiction A) solely financed by debt issue, i.e. by government borrowing. Absent any restrictions on how the two kinds of government spending are financed (by tax or debt), both jurisdictions would opt for complete debt financing. In the rest of this section we investigate the possibility of the first-best allocation being implemented – via Nash equilibrium – by imposing some restriction on the financing of government spending.

First, consider the restriction that no government borrowing is allowed in both jurisdictions. The analysis below shows that this financing restriction does not implement the first-best allocation when there is a positive probability of migration. In this case, the entire period-1 government spending in jurisdiction A, $G_1^A + G_2^A$ on the per-capita basis, is financed by a tax in period 1. Then, the expected lifetime utility of the representative individual in jurisdiction A becomes

$$u_1\left(y_1 - k^A - G_1^A - G_2^A\right) + g_1(G_1^A) + u_2\left(y_2 + k^A(1+r)\right) + (1-p)g_2(G_2^A) + pg_2(G_2^B). \quad (8)$$

The difference between (8) and (2) lies in that the total government spending $G_1^A + G_2^A$ is entirely financed by a tax in period 1 in (8), and entirely financed by debt in (2).

Maximizing (8), the first-order conditions with respect to k^A, G_1^A and G_2^A are respectively

$$-u_1'\left(y_1 - k^A - G_1^A - G_2^A\right) + (1+r)u_2'\left(y_2 + k^A(1+r)\right) = 0$$
$$-u_1'\left(y_1 - k^A - G_1^A - G_2^A\right) + g_1'(G_1^A) = 0 \qquad (9)$$
$$-u_1'\left(y_1 - k^A - G_1^A - G_2^A\right) + (1-p)g_2'(G_2^A) = 0.$$

Note that the best choices of k^A, G_1^A and G_2^A by jurisdiction A are independent of the choices by jurisdiction B, implying that the resulting Nash equilibrium is also a dominant strategy equilibrium.

It is obvious from comparing (9) with (4) and (6) that, when p = 0, the dominant strategy equilibrium allocation under the no-debt financing is identical to both the Nash equilibrium allocation under the all-debt financing and the first-best allocation, but when p > 0, the three allocations diverge from one another. Therefore, the presence of migration (and the externalities of one jurisdiction's fiscal policy on another) is not only the source of the suboptimal fiscal policies in the two jurisdictions, but also the source of the nonequivalence between debt and tax financing.

Next, consider the financing restriction that public consumption must be entirely financed by the contemporary tax. The analysis below shows that this financing restriction implements the first-best allocation when there is a positive probability of migration. In this case, the expected lifetime utility of the representative individual in jurisdiction A is

$$u_1\left(y_1 - k^A - G_1^A - G_2^A\right) + g_1(G_1^A) + (1-p)u_2\left(y_2 + k^A(1+r)\right) + (1-p)g_2(G_2^A)$$
$$+pu_2\left(y_2 + k^A(1+r) + G_2^A(1+r) - G_2^B(1+r)\right) + pg_2(G_2^B). \qquad (10)$$

The representative individual chooses k^A, G_1^A and G_2^A to maximize (10), taking G_2^B as given. The first-order conditions with respect to k^A, G_1^A and G_2^A are respectively

$$-u_1'\left(y_1-k^A-G_1^A-G_2^A\right)+(1+r)(1-p)u_2'\left(y_2+k^A(1+r)\right)+(1+r)pu_2'\left(y_2+k^A(1+r)+G_2^A(1+r)-G_2^B(1+r)\right)=0$$
$$-u_1'\left(y_1-k^A-G_1^A-G_2^A\right)+g_1'(G_1^A)=0$$
$$-u_1'\left(y_1-k^A-G_1^A-G_2^A\right)+(1-p)g_2'(G_2^A)+(1+r)pu_2'\left(y_2+k^A(1+r)+G_2^A(1+r)-G_2^B(1+r)\right)=0.$$

Imposing the symmetry conditions, that $k^A=k^B=k^*$, $G_1^A=G_1^B=G_1^*$ and $G_2^A=G_2^B=G_2^*$, and rearranging the three equations above, the symmetric Nash equilibrium (k^*, G_1^*, G_2^*) under the restriction of no-debt financing of public consumption is determined by the following equations:

$$\begin{aligned}&-u_1'\left(y_1-k^*-G_1^*-G_2^*\right)+(1+r)u_2'\left(y_2+k^*(1+r)\right)=0\\&-u_1'\left(y_1-k^*-G_1^*-G_2^*\right)+g_1'(G_1^*)=0\\&-u_1'\left(y_1-k^*-G_1^*-G_2^*\right)+g_2'(G_2^*)=0.\end{aligned}\tag{11}$$

Noting that (11) and (6) are identical, we immediately have the following proposition.

Proposition 2. The first-best allocation is achieved in the Nash equilibrium by imposing the restriction that public consumption must be financed by the contemporary tax.

It is important to note that the financing restriction that implements the first-best allocation only requires all of public consumption to be financed by the current tax, and does not further require that all of public investment be financed by debt. Nevertheless, requiring all of public consumption to be financed by current tax alone is sufficient for efficiency exactly because the equilibrium choice for the financing of public investment will exclusively be debt.

3. A Fiscal-Policy Game between Two Jurisdictions with One-Directional Migration

We modify the fiscal-policy game in the last section by allowing the two jurisdictions to be of different sizes and by imposing an additional assumption that migration is one-directional. A fiscal-policy game with these features can be used to analyze migration's effects on fiscal policies in situations where one of the two linked jurisdictions/countries is the destination of migration (e.g., the U.S., Australia or Canada) and the other is the origin of migration (e.g., Puerto Rico, Mexico, China or India).[10]

The initial population size of jurisdiction A (the migration destination) is normalized to 1 while that of jurisdiction B (the migration origin) is denoted by parameter $n \in (0, \infty)$. Assume that individuals in jurisdiction A will always stay, whereas individuals in jurisdiction B migrate to jurisdiction A, with probability p, in period 2.[11] As in the last section, individuals are assumed to be identical within and between jurisdictions

3.1 Optimal Fiscal Policy of Jurisdiction A (the Destination of Migration)

The first- and second-period consumptions of the representative individual born in jurisdiction A are, respectively, given by

$$\begin{aligned} c_1^A &= y_1 - k^A - G_1^A - G_2^A \\ c_2^A &= y_2 + k^A(1+r) + \alpha_A(G_1^A + G_2^A)(1+r) - \frac{\alpha_A(G_1^A + G_2^A)(1+r)}{1+np} \\ &= y_2 + k^A(1+r) + \frac{np}{1+np}\alpha_A(G_1^A + G_2^A)(1+r). \end{aligned} \tag{12}$$

In period 1, the representative individual in jurisdiction A decides on the amount invested in productive capital k^A, per-capita public consumption G_1^A, per-capita public investment G_2^A and the fraction of period-1 public spending that is financed by borrowing α_A to maximize

$$u_1(c_1^A) + g_1(G_1^A) + u_2(c_2^A) + g_2\left(\frac{G_2^A}{1+np}\right). \tag{13}$$

[10] As is demonstrated in this section, a game with these features turns out to be mathematically manageable, even though some derivations can be quite involved. In contrast, a more general fiscal-policy game with non-identical jurisdictions and two-way migration would be intractable because of the many interdependent endogenous variables.

[11] When n > 1, migration is from a country with a larger population to a country with a smaller population (e.g. from China to Canada or India to Australia). When n < 1, on the other hand, migration is from a country with a smaller population to a country with a larger population (e.g. from Puerto Rico or Mexico to the (Mainland) U.S.).

Note that migration from jurisdiction B has two opposing effects on Jurisdiction A. First, it helps pay for the debt retirement in jurisdiction A, lowering the tax burden born by the representative individual in jurisdiction A. This effect is reflected by the additional component in period-2 consumption, $\frac{np}{1+np}\alpha_A(G_1^A+G_2^A)(1+r)$, which would be zero if p = 0. Second, migration from jurisdiction B competes for period-2 public resources in jurisdiction A, as period-2 per-capita consumption of publicly-provided goods is $\frac{G_2^A}{1+np}$, which is smaller than G_2^A due to migration.

Because migration from jurisdiction B makes debt financing less costly than contemporary-tax financing in jurisdiction A, $\alpha_A = 1$ must be part of the optimal fiscal policy in jurisdiction A. Therefore, jurisdiction A's problem reduces to choosing k^A, G_1^A and G_2^A to maximize (13), subject to (12) and $\alpha_A = 1$.

The first-order conditions with respect to k^A, G_1^A and G_2^A are respectively

$$\begin{aligned}
&-u_1'\left(y_1-k^A-G_1^A-G_2^A\right)+(1+r)u_2'\left(y_2+k^A(1+r)+\frac{np}{1+np}(G_1^A+G_2^A)(1+r)\right)=0\\
&-u_1'\left(y_1-k^A-G_1^A-G_2^A\right)+g_1'(G_1^A)+(1+r)\left(\frac{np}{1+np}\right)u_2'\left(y_2+k^A(1+r)+\frac{np}{1+np}(G_1^A+G_2^A)(1+r)\right)=0\\
&-u_1'\left(y_1-k^A-G_1^A-G_2^A\right)+\left(\frac{1}{1+np}\right)g_2'\left(\frac{G_2^A}{1+np}\right)+(1+r)\left(\frac{np}{1+np}\right)u_2'\left(y_2+k^A(1+r)+\frac{np}{1+np}(G_1^A+G_2^A)(1+r)\right)=0.
\end{aligned} \tag{14}$$

It is straightforward to show that the corresponding Hessian matrix is negative definite, and therefore the second-order condition for the expected lifetime utility maximization problem is satisfied.

Note that from (14), (np) – the number of migrants to jurisdiction A relative to the size of its original population – can be treated as a single parameter in jurisdiction A's optimization problem. The proposition below states how jurisdiction A's optimal fiscal policy responses to changes in this parameter.

Proposition 3. The comparative statics of the optimal choices by jurisdiction A – k^A, G_1^A and G_2^A – with respect to the number of incoming migrants (np) are

$$\frac{dk^A}{d(np)} < 0, \frac{dG_1^A}{d(np)} > 0, \text{ and } \frac{d(G_1^A + G_2^A)}{d(np)} > 0.$$

Proof: See the appendix.

Proposition 3 says that as the number of incoming migrants increases, residents in jurisdiction A hold less productive capital ($\frac{dk^A}{d(np)} < 0$), spend more on public consumption ($\frac{dG_1^A}{d(np)} > 0$) and incur more debt to finance a larger total spending in period 1 ($\frac{d(G_1^A + G_2^A)}{d(np)} > 0$). The explanation for these results is straightforward. With more incoming migrants to help with debt retirement, jurisdiction A has an incentive to incur more debt ($\frac{d(G_1^A + G_2^A)}{d(np)} > 0$) to finance a larger amount of public consumption in period 1 ($\frac{dG_1^A}{d(np)} > 0$). Moreover, because people in jurisdiction A hold more government bonds which are a perfect substitute for productive capital, they hold less productive capital ($\frac{dk^A}{d(np)} < 0$).

These results for the migration destination are very similar to those for the two identical jurisdictions in a model of mutual migration stated in Proposition 1. This seems to suggest that the in-migration alone provides much of the explanation for the comparative statics results obtained within the model of mutual migration.

Nevertheless, one result in Proposition 1, namely $\frac{dG_2^*}{dp} < 0$, does not have a counterpart in Proposition 3. In other words, there does not exist a clear-cut result for the sign of $\frac{dG_2^A}{d(np)}$. The reason for this is also readily understandable. Whereas more incoming migrants make the costs of financing public investment G_2^A lower, they also further dilute the benefits from the public investment. These two effects work in opposite directions, rendering the sign of $\frac{dG_2^A}{d(np)}$ theoretically ambiguous.

3.2 Optimal Fiscal Policy of Jurisdiction B (the Origin of Migration)

The period-1 private consumption of the representative individual in jurisdiction B is

$$c_1^B = y_1 - k^B - G_1^B - G_2^B, \tag{15}$$

where k^B is the individual's investment in productive capital, G_1^B is per-capita public consumption and G_2^B is per-capita public investment, all in jurisdiction B.

The period-2 private consumption of the representative individual in jurisdiction B depends on whether the individual stays in jurisdiction B or migrates to jurisdiction A in period 2. If staying put in period 2 (with probability 1-p), the individual will pay for the debt repayment in jurisdiction B, resulting in his private consumption being

$$\begin{aligned} c_2^{B,S} &= y_2 + k^B(1+r) + \alpha_B(G_1^B + G_2^B)(1+r) - \frac{\alpha_B(G_1^B + G_2^B)(1+r)}{1-p} \\ &= y_2 + k^B(1+r) - \frac{p}{1-p}\alpha_B(G_1^B + G_2^B)(1+r) \end{aligned}, \tag{16}$$

where α_B is the fraction of period-1 public spending in jurisdiction B that is debt-financed.

If migrating to jurisdiction A in period 2 (with probability p), the representative individual will be subject to the government in jurisdiction A, paying taxes in period 2 to retire the debt incurred by jurisdiction A (noting that jurisdiction A optimally opts for $\alpha_A = 1$). Therefore, his private consumption in this case is

$$c_2^{B,M} = y_2 + k^B(1+r) + \alpha_B(G_1^B + G_2^B)(1+r) - \frac{(G_1^A + G_2^A)(1+r)}{1+np}. \tag{17}$$

Taking G_1^A and G_2^A as given, jurisdiction B chooses α_B, k^B, G_1^B and G_2^B to maximize

$$u_1(c_1^B) + g_1(G_1^B) + (1-p)\left[u_2(c_2^{B,S}) + g_2\left(\frac{G_2^B}{1-p}\right)\right] + p\left[u_2(c_2^{B,M}) + g_2\left(\frac{G_2^A}{1+np}\right)\right], \tag{18}$$

subject to (15) – (17). Note that, unlike the optimization problems in Section 2 and Subsection 3.1, the representative individual in jurisdiction B does not necessarily want $\alpha_B = 1$ because although there is a probability p for him to migrate to jurisdiction A, hence not being responsible for the debt retirement in jurisdiction B, there is also a probability 1–p for him to stay put, hence picking up the debt-retiring taxes that are left behind by those who migrated to jurisdiction A.

The first-order conditions with respect to α_B, k^B, G_1^B and G_2^B are respectively

$$\begin{aligned}
&-p(1+r)(G_1^B + G_2^B)u_2'\left(c_2^{B,S}\right) + p(1+r)(G_1^B + G_2^B)u_2'\left(c_2^{B,M}\right) = 0 \\
&-u_1'\left(c_1^B\right) + (1+r)(1-p)u_2'\left(c_2^{B,S}\right) + (1+r)pu_2'\left(c_2^{B,M}\right) = 0 \\
&-u_1'\left(c_1^B\right) + g_1'\left(G_1^B\right) - (1+r)p\alpha_B u_2'\left(c_2^{B,S}\right) + (1+r)p\alpha_B u_2'\left(c_2^{B,M}\right) = 0 \\
&-u_1'\left(c_1^B\right) + g_2'\left(\frac{G_2^B}{1-p}\right) - (1+r)p\alpha_B u_2'\left(c_2^{B,S}\right) + (1+r)p\alpha_B u_2'\left(c_2^{B,M}\right) = 0.
\end{aligned} \tag{19}$$

It is straightforward to show that the corresponding Hessian matrix is negative definite, and therefore the second-order condition for the expected lifetime utility maximization problem is satisfied.

The first equation in (19) implies $c_2^{B,S} = c_2^{B,M}$. So (19) can be simplified to

$$\begin{aligned}
&\alpha_B(G_1^B + G_2^B) - \frac{1-p}{1+np}(G_1^A + G_2^A) = 0 \\
&-u_1'\left(y_1 - k^B - G_1^B - G_2^B\right) + (1+r)u_2'\left(y_2 + k^B(1+r) - \frac{p}{1-p}\alpha_B(G_1^B + G_2^B)(1+r)\right) = 0 \\
&-u_1'\left(y_1 - k^B - G_1^B - G_2^B\right) + g_1'\left(G_1^B\right) = 0 \\
&-u_1'\left(y_1 - k^B - G_1^B - G_2^B\right) + g_2'\left(\frac{G_2^B}{1-p}\right) = 0,
\end{aligned} \tag{20}$$

which is the set of equations that fully determine the optimal α_B, k^B, G_1^B and G_2^B, where $G_1^A + G_2^A$ and other parameters (*n* and *p* in particular) are taken as given.

Importantly, parameters *p* and *n* have impacts on the optimal α_B, k^B, G_1^B and G_2^B also through $G_1^A + G_2^A$. Therefore, migration's effects on the fiscal policies of jurisdiction B may depend on how responsive jurisdiction A's fiscal policies are to migration. The following propositions about the comparative statics results for jurisdiction B indicate that this is indeed the case. We first give the proposition about the comparative statics with respect to *p*.

Proposition 4. The comparative statics of jurisdiction B's optimal choices with respect to *p* are characterized as follows: If

$$\frac{d\left(G_1^A + G_2^A\right)}{dp}\left(\frac{p}{G_1^A + G_2^A}\right) \geq \left(\frac{p}{1-p}\right)\left(\frac{1+n}{1+np}\right), \tag{21}$$

then $\frac{d\alpha_B}{dp} > 0$, $\frac{dk^B}{dp} > 0$, $\frac{dG_2^B}{dp} < 0$, and $\frac{d\left[\alpha_B(G_1^B + G_2^B)\right]}{dp} \geq 0$.

Proof: See the appendix.

The left-hand side of (21) is the elasticity of $G_1^A + G_2^A$ with respect to *p*, and the right-hand side of (21) is increasing in both *p* and *n*. So condition (21) simply requires the elasticity of $G_1^A + G_2^A$ with respect to *p* to be large enough relatively to the level of migration (the larger the *p* or n, the higher the level of migration). Condition (21) highlights the role played by jurisdiction A's response to migration in determining the optimal response to migration by jurisdiction B.

While condition (21) is only sufficient, not necessary, for the comparative statics results in Proposition 4, it is both sufficient and necessary for the result of $\frac{d\left[\alpha_B(G_1^B+G_2^B)\right]}{dp}\geq 0$. In other words, condition (21) is identical to requiring that the debt level in jurisdiction B increase as the probability of migration increases.

We now give the proposition about the comparative statics with respect to n.

Proposition 5. The comparative statics of jurisdiction B's optimal choices with respect to n are characterized as follows: $\frac{d\alpha_B}{dn}>0$, $\frac{dk^B}{dn}>0$, $\frac{dG_1^B}{dn}<0$, $\frac{dG_2^B}{dn}<0$, and $\frac{d\left[\alpha_B(G_1^B+G_2^B)\right]}{dn}>0$ if and only if

$$\frac{d\left(G_1^A+G_2^A\right)}{dn}\left(\frac{n}{G_1^A+G_2^A}\right)>\frac{np}{1+np}. \tag{22}$$

Proof: See the appendix.

The left-hand side of (22) is the elasticity of $G_1^A+G_2^A$ with respect to n, and the right-hand side of (22) is increasing in np. So condition (22) simply requires the elasticity of $G_1^A+G_2^A$ with respect to n to be large enough relatively to the level of migration (the larger the np, the higher the level of migration). Note that condition (22) is both sufficient and necessary. So the optimal responses of jurisdiction B to an increase in n critically depend on how responsive jurisdiction A is to the increase in n.

4. Generalizations to the Non-Separable Utility Functions, Durable Goods, and Cross-Jurisdiction Debt Holding

In the last two sections, we assumed that the utility functions are separable in the private consumption and the publicly-provided good in the same period, that there is a perfect dichotomy between public consumption and public investment, and that the debt issued by a jurisdiction is exclusively held by the residents of that jurisdiction, in order to simplify the technical aspect of the analysis and to gain sharp insights. In this section, we generalize the benchmark analysis in Section 2 of a fiscal-policy game between two identical jurisdictions by allowing the utility functions to be non-separable, some government spending to simultaneously provide public consumption and public investment (i.e. the case of durable publicly-provided goods), or some share of one jurisdiction's debt to be held by the residents of the other jurisdiction.

4.1. Non-Separable Utility Functions

We replace the lifetime utility function of (1) with

$$u(c_1, G_1) + v(c_2, G_2). \tag{1'}$$

Note that this lifetime utility function includes (1) as a special case in which $u_{cG} = v_{cG} = 0$. Moreover, it also includes the often-used lifetime utility function, $u(c_1, G_1) + \beta u(c_2, G_2)$, as a special case where $v(c_2, G_2) = \beta u(c_2, G_2)$.[12] Functions u and v are assumed to be twice differentiable, strictly increasing in both variables (i.e. $u_c, u_G, v_c, v_G > 0$) and strictly concave (i.e. $u_{cc}, u_{GG}, v_{cc}, v_{GG} < 0$, $u_{cc}u_{GG} - (u_{cG})^2 > 0$ and $v_{cc}v_{GG} - (v_{cG})^2 > 0$). In addition, we assume that c and G are weakly complements in both u and v (i.e. $u_{cG}, v_{cG} \geq 0$).[13]

---

[12] β is typically treated as a preference parameter with more patient individuals having a larger value of β, but it may also include a longevity component in which a lower mortality rate corresponds to a larger value of β.

[13] The assumption of complementarity between c and G is somewhat restrictive because it does not allow the two goods to be substitutes. On the other hand, this assumption is satisfied by most often-used utility functions and greatly simplifies the technical aspect of the analysis in this section.

Taking G_1^B, G_2^B and α_B as given, the representative individual in jurisdiction A chooses k^A, G_1^A, G_2^A and α_A to maximize

$$u(c_1^A, G_1^A) + (1-p)v(c_2^{A,S}, G_2^A) + pv(c_2^{A,M}, G_2^B), \tag{2'}$$

where

$$\begin{aligned}
c_1^A &= y_1 - k^A - G_1^A - G_2^A \\
c_2^{A,S} &= y_2 + k^A(1+r) + \alpha_A(G_1^A + G_2^A)(1+r) - \alpha_A(G_1^A + G_2^A)(1+r) = y_2 + k^A(1+r) \\
c_2^{A,M} &= y_2 + k^A(1+r) + \alpha_A(G_1^A + G_2^A)(1+r) - \alpha_B(G_1^B + G_2^B)(1+r).
\end{aligned}$$

Obviously, the individual's optimal choices should include $\alpha_A = 1$. Therefore, the above lifetime utility maximization problem can be reduced to a maximization problem with respect to k^A, G_1^A and G_2^A, taking G_1^B and G_2^B as given and letting $\alpha_A = \alpha_B = 1$. The first-order conditions with respect to k^A, G_1^A and G_2^A are respectively

$$\begin{aligned}
&-u_c\left(y_1 - k^A - G_1^A - G_2^A, G_1^A\right) + (1+r)(1-p)v_c\left(y_2 + k^A(1+r), G_2^A\right) \\
&\qquad + (1+r)pv_c\left(y_2 + k^A(1+r) + (G_1^A + G_2^A)(1+r) - (G_1^B + G_2^B)(1+r), G_2^B\right) = 0 \\
&-u_c\left(y_1 - k^A - G_1^A - G_2^A, G_1^A\right) + u_G\left(y_1 - k^A - G_1^A - G_2^A, G_1^A\right) \\
&\qquad + (1+r)pv_c\left(y_2 + k^A(1+r) + (G_1^A + G_2^A)(1+r) - (G_1^B + G_2^B)(1+r), G_2^B\right) = 0 \\
&-u_c\left(y_1 - k^A - G_1^A - G_2^A, G_1^A\right) + (1-p)v_G\left(y_2 + k^A(1+r), G_2^A\right) \\
&\qquad + (1+r)pv_c\left(y_2 + k^A(1+r) + (G_1^A + G_2^A)(1+r) - (G_1^B + G_2^B)(1+r), G_2^B\right) = 0.
\end{aligned} \tag{3'}$$

It is straightforward to show that the corresponding Hessian matrix is negative definite, and therefore the second-order condition for the expected lifetime utility maximization problem is satisfied.

Imposing the symmetry conditions, that $k^A = k^B = k^*, G_1^A = G_1^B = G_1^*$ and $G_2^A = G_2^B = G_2^*$, and rearranging the three equations in (3'), the symmetric Nash equilibrium (k^*, G_1^*, G_2^*) is determined by the following equations:

$$
\begin{aligned}
-u_c\left(y_1-k^*-G_1^*-G_2^*,G_1^*\right)+(1+r)v_c\left(y_2+k^*(1+r),G_2^*\right)=0 \\
-(1-p)u_c\left(y_1-k^*-G_1^*-G_2^*,G_1^*\right)+u_G\left(y_1-k^*-G_1^*-G_2^*,G_1^*\right)=0 \\
-u_c\left(y_1-k^*-G_1^*-G_2^*,G_1^*\right)+v_G\left(y_2+k^*(1+r),G_2^*\right)=0.
\end{aligned}
\tag{4'}
$$

Proposition 1'. Under the stated assumptions on non-separable utility functions u and v, the comparative statics of the symmetric Nash equilibrium (k^*, G_1^*, G_2^*) with respect to p are still characterized by $\frac{dk^*}{dp}<0, \frac{dG_1^*}{dp}>0$ and $\frac{dG_2^*}{dp}<0$. In addition, $\frac{d(G_1^*+G_2^*)}{dp}>0$ if $u_{cc}\left[(1+r)v_{cG}-v_{GG}\right]<(1+r)u_{cG}\left[(1+r)v_{cc}-v_{cG}\right]$, which is particularly true when $u_{cG}=0$.

Proof: See the appendix.

Now we demonstrate that Proposition 2 in Section 2 can also be generalized to the case of non-separable utility functions. In the more general case, the first-best allocation in both jurisdictions, denoted $(\hat{k}, \hat{G}_1, \hat{G}_2)$, maximizes

$$
u\left(y_1-k-G_1-G_2,G_1\right)+v\left(y_2+k(1+r),G_2\right). \tag{5'}
$$

$(\hat{k}, \hat{G}_1, \hat{G}_2)$ is thus determined by the following first-order conditions:

$$
\begin{aligned}
&-u_c\left(y_1-\hat{k}-\hat{G}_1-\hat{G}_2,\hat{G}_1\right)+(1+r)v_c\left(y_2+\hat{k}(1+r),\hat{G}_2\right)=0 \\
&-u_c\left(y_1-\hat{k}-\hat{G}_1-\hat{G}_2,\hat{G}_1\right)+u_G\left(y_1-\hat{k}-\hat{G}_1-\hat{G}_2,\hat{G}_1\right)=0 \\
&-u_c\left(y_1-\hat{k}-\hat{G}_1-\hat{G}_2,\hat{G}_1\right)+v_G\left(y_2+\hat{k}(1+r),\hat{G}_2\right)=0.
\end{aligned}
\tag{6'}
$$

Consider the effect on the symmetric Nash equilibrium from the financing restriction that public consumption must be entirely financed by the contemporary tax. In this case, the expected lifetime utility of the representative individual in jurisdiction A is

$$
\begin{aligned}
&u\left(y_1-k^A-G_1^A-G_2^A,G_1^A\right)+(1-p)v\left(y_2+k^A(1+r),G_2^A\right) \\
&+pv\left(y_2+k^A(1+r)+G_2^A(1+r)-G_2^B(1+r),G_2^B\right).
\end{aligned}
\tag{10'}
$$

The representative individual chooses k^A, G_1^A and G_2^A to maximize (10'), taking G_2^B as given. The first-order conditions with respect to k^A, G_1^A and G_2^A are respectively

$$
\begin{aligned}
&-u_c\left(y_1 - k^A - G_1^A - G_2^A, G_1^A\right) + (1+r)(1-p)v_c\left(y_2 + k^A(1+r), G_2^A\right) \\
&\qquad + (1+r)pv_c\left(y_2 + k^A(1+r) + G_2^A(1+r) - G_2^B(1+r), G_2^B\right) = 0 \\
&-u_c\left(y_1 - k^A - G_1^A - G_2^A, G_1^A\right) + u_G\left(y_1 - k^A - G_1^A - G_2^A, G_1^A\right) = 0 \\
&-u_c\left(y_1 - k^A - G_1^A - G_2^A, G_1^A\right) + (1-p)v_G\left(y_2 + k^A(1+r), G_2^A\right) \\
&\qquad + (1+r)pv_c\left(y_2 + k^A(1+r) + G_2^A(1+r) - G_2^B(1+r), G_2^B\right) = 0.
\end{aligned}
$$

Imposing the symmetry conditions, that $k^A = k^B = k^*$, $G_1^A = G_1^B = G_1^*$ and $G_2^A = G_2^B = G_2^*$, and rearranging the three equations above, the symmetric Nash equilibrium (k^*, G_1^*, G_2^*) under the restriction of no-debt financing of public consumption is determined by the following equations:

$$
\begin{aligned}
&-u_c\left(y_1 - k^* - G_1^* - G_2^*, G_1^*\right) + (1+r)v_c\left(y_2 + k^*(1+r), G_2^*\right) = 0 \\
&-u_c\left(y_1 - k^* - G_1^* - G_2^*, G_1^*\right) + u_G\left(y_1 - k^* - G_1^* - G_2^*, G_1^*\right) = 0 \qquad (11') \\
&-u_c\left(y_1 - k^* - G_1^* - G_2^*, G_1^*\right) + v_G\left(y_2 + k^*(1+r), G_2^*\right) = 0.
\end{aligned}
$$

Noting that (11') and (6') are identical, we immediately have the following proposition.

Proposition 2'. For the more general case with non-separable utility functions u and v, the first-best allocation is achieved in the Nash equilibrium by imposing the restriction that public consumption must be financed by the contemporary tax.

4.2. Durable Publicly-Provided Goods

A main feature of this paper is the distinction between spending on public consumption and spending on public investment. The critical role played by this distinction is apparent in Propositions 1 and 2 of Section 2. Proposition 1 says that, among other things, as mutual migration intensifies, equilibrium public consumption increases and equilibrium public investment decreases; Proposition 2 says that under the restriction that public consumption is exclusively financed by the contemporary tax, the first-best allocation is implemented through the Nash equilibrium.

A question then arises as to whether these two propositions can be generalized to the case where some government spending simultaneously provides public consumption and public investment, i.e., the case of durable publicly-provided goods.

In this case, the representative individual in jurisdiction A chooses k^A, G_1^A and G_2^A – and for the same previously discussed reason the total spending $G_1^A + G_2^A$ is financed by debt – to maximize his expected lifetime utility

$$\begin{aligned} &u_1\left(y_1 - k^A - G_1^A - G_2^A\right) + g_1(G_1^A) + (1-p)u_2\left(y_2 + k^A(1+r)\right) + (1-p)g_2(\theta G_1^A + G_2^A) \\ &+ pu_2\left(y_2 + k^A(1+r) + (G_1^A + G_2^A)(1+r) - (G_1^B + G_2^B)(1+r)\right) + pg_2(\theta G_1^B + G_2^B), \end{aligned} \quad (2")$$

taking G_1^B and G_2^B as given. In (2"), government spending G_1 produces both public consumption and public investment, and $\theta \in [0,1)$ measures the durability of publicly-provided good G_1.[14] Obviously, (2) in Section 2 is a special case of (2") here where $\theta = 0$.

From the same derivations as those in Section 2, the symmetric Nash equilibrium (k^*, G_1^*, G_2^*) is found to satisfy:

$$\begin{aligned} &-u_1'\left(y_1 - k^* - G_1^* - G_2^*\right) + (1+r)u_2'\left(y_2 + k^*(1+r)\right) = 0 \\ &-(1-p)u_1'\left(y_1 - k^* - G_1^* - G_2^*\right) + g_1'(G_1^*) + \theta(1-p)g_2'(\theta G_1^* + G_2^*) = 0 \\ &-u_1'\left(y_1 - k^* - G_1^* - G_2^*\right) + g_2'(\theta G_1^* + G_2^*) = 0. \end{aligned} \quad (4")$$

Parallel to Proposition 1, we obtain the following result, the proof of which is omitted because it is similar to that of Proposition 1.

---

[14]Note that G_2 is strictly dominated by G_1 if $\theta \geq 1$. So we only consider $\theta \in [0,1)$ to give a role to G_2.

Proposition 1". The comparative statics of the symmetric Nash equilibrium (k^*, G_1^*, G_2^*) with respect to p are $\frac{dk^*}{dp} < 0, \frac{dG_1^*}{dp} > 0, \frac{d(\theta G_1^* + G_2^*)}{dp} < 0$ and $\frac{d(G_1^* + G_2^*)}{dp} > 0$.

Note that $\frac{d(\theta G_1^* + G_2^*)}{dp} < 0$ implies $\frac{dG_2^*}{dp} < 0$, given $\frac{dG_1^*}{dp} > 0$. The result $\frac{d(\theta G_1^* + G_2^*)}{dp} < 0$ is easy to understand once it is recognized that $\theta G_1^* + G_2^*$ is the total amount of publicly-provided good in period 2.

We next establish the parallel result to Proposition 2, for the case of durable G_1. The first-best allocation, denoted $(\hat{k}, \hat{G}_1, \hat{G}_2)$, maximizes

$$u_1\left(y_1 - k - G_1 - G_2\right) + g_1(G_1) + u_2\left(y_2 + k(1+r)\right) + g_2(\theta G_1 + G_2). \tag{5''}$$

The first-order conditions are:

$$\begin{aligned}
&-u_1'\left(y_1 - \hat{k} - \hat{G}_1 - \hat{G}_2\right) + (1+r)u_2'\left(y_2 + \hat{k}(1+r)\right) = 0 \\
&-u_1'\left(y_1 - \hat{k} - \hat{G}_1 - \hat{G}_2\right) + g_1'(\hat{G}_1) + \theta g_2'(\theta\hat{G}_1 + \hat{G}_2) = 0 \\
&-u_1'\left(y_1 - \hat{k} - \hat{G}_1 - \hat{G}_2\right) + g_2'(\theta\hat{G}_1 + \hat{G}_2) = 0.
\end{aligned} \tag{6''}$$

In order to implement the first-best allocation through the Nash play, consider the financing restriction in both jurisdictions that a portion of government spending on G_1, $(1-\theta)G_1$, must be financed by the contemporary tax. The analysis below shows that this financing restriction implements the first-best allocation. In this case, the expected lifetime utility of the representative individual in jurisdiction A is

$$\begin{aligned}
&u_1\left(y_1 - k^A - G_1^A - G_2^A\right) + g_1(G_1^A) + (1-p)u_2\left(y_2 + k^A(1+r)\right) + (1-p)g_2(\theta G_1^A + G_2^A) \\
&+ pu_2\left(y_2 + k^A(1+r) + (\theta G_1^A + G_2^A)(1+r) - (\theta G_1^B + G_2^B)(1+r)\right) + pg_2(\theta G_1^B + G_2^B).
\end{aligned} \tag{10''}$$

It can be shown that the symmetric Nash equilibrium, denoted (k^*, G_1^*, G_2^*), is determined by the following conditions:

$$\begin{aligned} &-u_1'\left(y_1-k^*-G_1^*-G_2^*\right)+(1+r)u_2'\left(y_2+k^*(1+r)\right)=0 \\ &-(1-\theta)u_1'\left(y_1-k^*-G_1^*-G_2^*\right)+g_1'(G_1^*)=0 \\ &-u_1'\left(y_1-k^*-G_1^*-G_2^*\right)+g_2'(\theta G_1^*+G_2^*)=0. \end{aligned} \qquad (11")$$

Noting that (11") and (6") are equivalent, we immediately have the following proposition.

Proposition 2". The first-best allocation is achieved in the Nash equilibrium by imposing the restriction that a portion of government spending on G_1, $(1-\theta)G_1$, must be financed by the contemporary tax.

One implication from comparing Proposition 2 and Proposition 2" is that the public financing institution that is able to implement the first-best allocation under non-durable publicly-provided goods is different from that under durable goods. The durability of publicly-provided good G_1 makes spending on G_1 simultaneously produce public consumption (which is G_1) and public investment (which is θG_1). An efficient public financing institution only requires that a portion of spending on G_1, $(1-\theta)G_1$ to be specific, be financed by the current tax.

4.3. Cross-Jurisdiction Debt Holding

Again suppose that jurisdiction A can finance its total spending in period 1, $G_1^A + G_2^A$, by a mix of tax collections or debt finance, and that α_A is the portion of the total spending that is financed by debt. However, instead of assuming that the per-capita debt amount $\alpha_A\left(G_1^A+G_2^A\right)$ is entirely held by the representative individual in jurisdiction A, we now assume that only some share λ $(0 \le \lambda \le 1)$ of this debt is held by the representative individual. Then the representative individual in jurisdiction A holds bonds issued by jurisdiction A in the amount of $\lambda\alpha_A\left(G_1^A + G_2^A\right)$ and bonds issued by jurisdiction B in the amount of $(1-\lambda)\alpha_B\left(G_1^B + G_2^B\right)$. The representative individual's private consumption in period 1 is therefore

$$c_1^A = y_1 - k^A - (1-\alpha_A)(G_1^A + G_2^A) - \lambda\alpha_A(G_1^A + G_2^A) - (1-\lambda)\alpha_B(G_1^B + G_2^B).$$

If staying put in period 2 (with probability 1-p), the representative individual will pay for the debt repayment in jurisdiction A, and also receive utility from the public investment in jurisdiction A. In particular, his private consumption is

$$c_2^{A,S} = y_2 + k^A(1+r) + \lambda\alpha_A(G_1^A + G_2^A)(1+r) + (1-\lambda)\alpha_B(G_1^B + G_2^B)(1+r) - \alpha_A(G_1^A + G_2^A)(1+r),$$

where r is the rate of return earned on productive capital as well as government bonds.

If migrating from jurisdiction A to B in period 2 (with probability p), the representative individual will be subject to the government in jurisdiction B, paying taxes in period 2 for the debt incurred by jurisdiction B, and receiving benefit from public investment that was undertaken in jurisdiction B. In particular, his private consumption is

$$c_2^{A,M} = y_2 + k^A(1+r) + \lambda\alpha_A(G_1^A + G_2^A)(1+r) + (1-\lambda)\alpha_B(G_1^B + G_2^B)(1+r) - \alpha_B(G_1^B + G_2^B)(1+r).$$

Note that regardless of what jurisdiction B chooses (including the value of α_B), and regardless of the values of k^A, G_1^A and G_2^A, the representative individual's $c_2^{A,M}$ is maximized when $\alpha_A = 1$, whereas the constraint for the net present value of c_1^A and $c_2^{A,S}$, or $c_1^A + \frac{c_2^{A,S}}{1+r}$, remains $y_1 - (G_1^A + G_2^A) + \frac{y_2}{1+r}$, which is independent of α_A. Therefore, in period 1 the representative individual in jurisdiction A will always choose to finance the entire total spending in period 1 with debt, that is $\alpha_A = 1$. The same argument leads to $\alpha_B = 1$.

Therefore, the representative individual in jurisdiction A chooses k^A, G_1^A and G_2^A to maximize his expected lifetime utility

$$\begin{aligned}&u_1\left(y_1-k^A-\lambda(G_1^A+G_2^A)-(1-\lambda)(G_1^B+G_2^B)\right)+g_1(G_1^A)\\&+(1-p)u_2\left(y_2+k^A(1+r)-(1-\lambda)(G_1^A+G_2^A)(1+r)+(1-\lambda)(G_1^B+G_2^B)(1+r)\right)+(1-p)g_2(G_2^A)\\&+pu_2\left(y_2+k^A(1+r)+\lambda(G_1^A+G_2^A)(1+r)-\lambda(G_1^B+G_2^B)(1+r)\right)+pg_2(G_2^B),\end{aligned} \tag{2'''}$$

taking G_1^B and G_2^B as given. The first-order conditions with respect to k^A, G_1^A and G_2^A are respectively

$$\begin{aligned}&-u_1'\left(y_1-k^A-\lambda(G_1^A+G_2^A)-(1-\lambda)(G_1^B+G_2^B)\right)+(1+r)(1-p)u_2'\left(y_2+k^A(1+r)-(1-\lambda)(G_1^A+G_2^A)(1+r)+(1-\lambda)(G_1^B+G_2^B)(1+r)\right)\\&\qquad+(1+r)pu_2'\left(y_2+k^A(1+r)+\lambda(G_1^A+G_2^A)(1+r)-\lambda(G_1^B+G_2^B)(1+r)\right)=0\\&-\lambda u_1'\left(y_1-k^A-\lambda(G_1^A+G_2^A)-(1-\lambda)(G_1^B+G_2^B)\right)+g_1'(G_1^A)\\&\qquad-(1+r)(1-p)(1-\lambda)u_2'\left(y_2+k^A(1+r)-(1-\lambda)(G_1^A+G_2^A)(1+r)+(1-\lambda)(G_1^B+G_2^B)(1+r)\right)\\&\qquad+(1+r)p\lambda u_2'\left(y_2+k^A(1+r)+\lambda(G_1^A+G_2^A)(1+r)-\lambda(G_1^B+G_2^B)(1+r)\right)=0\\&-\lambda u_1'\left(y_1-k^A-\lambda(G_1^A+G_2^A)-(1-\lambda)(G_1^B+G_2^B)\right)-(1+r)(1-p)(1-\lambda)u_2'\left(y_2+k^A(1+r)-(1-\lambda)(G_1^A+G_2^A)(1+r)+(1-\lambda)(G_1^B+G_2^B)(1+r)\right)\\&\qquad+(1-p)g_2'(G_2^A)+(1+r)p\lambda u_2'\left(y_2+k^A(1+r)+\lambda(G_1^A+G_2^A)(1+r)-\lambda(G_1^B+G_2^B)(1+r)\right)=0.\end{aligned} \tag{3'''}$$

Imposing the symmetry conditions, that $k^A=k^B=k^*$, $G_1^A=G_1^B=G_1^*$ and $G_2^A=G_2^B=G_2^*$, and rearranging the three equations in (3'''), the symmetric Nash equilibrium (k^*, G_1^*, G_2^*) is determined by the following equations:

$$\begin{aligned}&-u_1'\left(y_1-k^*-G_1^*-G_2^*\right)+(1+r)u_2'\left(y_2+k^*(1+r)\right)=0\\&-(1-p)u_1'\left(y_1-k^*-G_1^*-G_2^*\right)+g_1'(G_1^*)=0\\&-u_1'\left(y_1-k^*-G_1^*-G_2^*\right)+g_2'(G_2^*)=0.\end{aligned} \tag{4'''}$$

Note that (4''') is identical to (4). That is, the Nash equilibrium is characterized by the same set of conditions regardless of the value of λ, the share of the debt issued by a jurisdiction that is held by the residents of the same jurisdiction.

5. Concluding Summary

We construct a two-jurisdiction, two-period model to study the effects of the between-jurisdiction migration on the fiscal policies of local governments. In period 1, each jurisdiction decides how much to spend on public consumption and how much on public investment. Each jurisdiction can finance this total spending with a mix of debt and contemporary taxation. In period 2, a fixed portion of the residents in each jurisdiction are randomly selected to migrate to another jurisdiction, and, as a result, both receive benefits from public investment in the new jurisdiction and also incur responsibility for the debt repayment in the new jurisdiction.

Two main findings are obtained in this context. First, as the level of migration increases, both the spending on public consumption and total spending (which is 100% financed by government borrowing in the Nash equilibrium) increase, whereas spending on public investment decreases. The implication is that increased human mobility is another reason for excessive government debt. Second, the first-best allocation can be obtained as a Nash equilibrium outcome by imposing the restriction that public consumption should be exclusively financed with contemporary taxation. This provides a theoretical justification for various national governments imposing debt limits on local governments.

The paper also considers a model with one-directional migration and obtains results on how migration affects the fiscal policies of both the jurisdiction of migration destination and the jurisdiction of migration origin. It finds that migration's effects on the migration destination are very similar to those obtained in the model with mutual migration, and that migration's effects on the migration origin critically depend on how responsive the fiscal policies of the migration destination are to migration.

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Appendix

Proof of Proposition 1

Differentiating the three equations in (4) with respect to p, respectively, taking k^*, G_1^*, G_2^* as functions of p, yields

$$\left[u_1'' + (1+r)^2 u_2''\right]\frac{dk^*}{dp} + u_1''\frac{dG_1^*}{dp} + u_1''\frac{dG_2^*}{dp} = 0$$

$$(1-p)u_1''\frac{dk^*}{dp} + \left[(1-p)u_1'' + g_1''\right]\frac{dG_1^*}{dp} + (1-p)u_1''\frac{dG_2^*}{dp} = -u_1' .$$

$$u_1''\frac{dk^*}{dp} + u_1''\frac{dG_1^*}{dp} + (u_1'' + g_2'')\frac{dG_2^*}{dp} = 0$$

We have

$$\Delta \equiv \begin{vmatrix} u_1'' + (1+r)^2 u_2'' & u_1'' & u_1'' \\ (1-p)u_1'' & (1-p)u_1'' + g_1'' & (1-p)u_1'' \\ u_1'' & u_1'' & u_1'' + g_2'' \end{vmatrix}$$

$$= u_1'' g_1''\left[(1+r)^2 u_2'' + g_2''\right] + u_2'' g_2''(1+r)^2\left[(1-p)u_1'' + g_1''\right] < 0.$$

Therefore,

$$\frac{dk^*}{dp} = \frac{1}{\Delta}\begin{vmatrix} 0 & u_1'' & u_1'' \\ -u_1' & (1-p)u_1'' + g_1'' & (1-p)u_1'' \\ 0 & u_1'' & u_1'' + g_2'' \end{vmatrix} = \frac{u_1'}{\Delta}\left(u_1'' g_2''\right) < 0,$$

$$\frac{dG_1^*}{dp} = \frac{1}{\Delta}\begin{vmatrix} u_1'' + (1+r)^2 u_2'' & 0 & u_1'' \\ (1-p)u_1'' & -u_1' & (1-p)u_1'' \\ u_1'' & 0 & u_1'' + g_2'' \end{vmatrix} = \frac{-u_1'}{\Delta}\left[(1+r)^2 u_1'' u_2'' + u_1'' g_2'' + (1+r)^2 u_2'' g_2''\right] > 0,$$

$$\frac{dG_2^*}{dp} = \frac{1}{\Delta}\begin{vmatrix} u_1'' + (1+r)^2 u_2'' & u_1'' & 0 \\ (1-p)u_1'' & (1-p)u_1'' + g_1'' & -u_1' \\ u_1'' & u_1'' & 0 \end{vmatrix} = \frac{u_1'}{\Delta}\left[(1+r)^2 u_1'' u_2''\right] < 0,$$

$$\frac{d(G_1^* + G_2^*)}{dp} = \frac{-u_1'}{\Delta}\left[u_1'' g_2'' + (1+r)^2 u_2'' g_2''\right] > 0.$$

Proof of Proposition 3

Rearranging the equations in (14), we have

$$-u_1'\left(y_1-k^A-G_1^A-G_2^A\right)+(1+r)u_2'\left(y_2+k^A(1+r)+\frac{np}{1+np}(G_1^A+G_2^A)(1+r)\right)=0$$

$$-\left(\frac{1}{1+np}\right)u_1'\left(y_1-k^A-G_1^A-G_2^A\right)+g_1'(G_1^A)=0$$

$$-u_1'\left(y_1-k^A-G_1^A-G_2^A\right)+g_2'\left(\frac{G_2^A}{1+np}\right)=0.$$

Differentiating the above equations with respect to (np), respectively, yields

$$\left[u_1''+(1+r)^2u_2''\right]\frac{dk^A}{d(np)}+\left[u_1''+\frac{np(1+r)^2}{1+np}u_2''\right]\frac{dG_1^A}{d(np)}+\left[u_1''+\frac{np(1+r)^2}{1+np}u_2''\right]\frac{dG_2^A}{d(np)}=-\frac{(1+r)^2}{(1+np)^2}(G_1^A+G_2^A)u_2''$$

$$\left(\frac{u_1''}{1+np}\right)\frac{dk^A}{d(np)}+\left(\frac{u_1''}{1+np}+g_1''\right)\frac{dG_1^A}{d(np)}+\left(\frac{u_1''}{1+np}\right)\frac{dG_2^A}{d(np)}=-\frac{1}{(1+np)^2}u_1'$$

$$u_1''\frac{dk^A}{d(np)}+u_1''\frac{dG_1^A}{d(np)}+\left(u_1''+\frac{g_2''}{1+np}\right)\frac{dG_2^A}{d(np)}=\frac{G_2^A}{(1+np)^2}g_2''.$$

We have

$$\Delta^A\equiv\begin{vmatrix} u_1''+(1+r)^2u_2'' & u_1''+\frac{np(1+r)^2}{1+np}u_2'' & u_1''+\frac{np(1+r)^2}{1+np}u_2'' \\ \frac{u_1''}{1+np} & \frac{u_1''}{1+np}+g_1'' & \frac{u_1''}{1+np} \\ u_1'' & u_1'' & u_1''+\frac{g_2''}{1+np} \end{vmatrix}$$

$$=\left(\frac{1}{1+np}\right)\left\{g_1''g_2''\left[u_1''+(1+r)^2u_2''\right]+(1+r)^2u_1''u_2''\left[g_1''+\left(\frac{1}{1+np}\right)^2g_2''\right]\right\}<0.$$

Therefore,

$$\frac{dk^A}{d(np)}=\frac{1}{\Delta^A}\left|\begin{array}{ccc} -\dfrac{(1+r)^2}{(1+np)^2}(G_1^A+G_2^A)u_2'' & u_1''+\dfrac{np(1+r)^2}{1+np}u_2'' & u_1''+\dfrac{np(1+r)^2}{1+np}u_2'' \\ -\dfrac{1}{(1+np)^2}u_1' & \dfrac{u_1''}{1+np}+g_1'' & \dfrac{u_1''}{1+np} \\ \dfrac{G_2^A}{(1+np)^2}g_2'' & u_1'' & u_1''+\dfrac{g_2''}{1+np} \end{array}\right|$$

$$=\frac{1}{\Delta^A}\left\{\begin{array}{l} -\dfrac{(1+r)^2}{(1+np)^2}(G_1^A+G_2^A)u_2''\left[\left(\dfrac{1}{1+np}\right)^2 u_1''g_2''+u_1''g_1''+\dfrac{g_1''g_2''}{1+np}\right] \\ +\dfrac{g_2''}{(1+np)^2}\left(\dfrac{u_1'}{1+np}-G_2^A g_1''\right)\left[u_1''+\dfrac{np(1+r)^2}{1+np}u_2''\right] \end{array}\right\}<0,$$

$$\frac{dG_1^A}{d(np)}=\frac{1}{\Delta^A}\left|\begin{array}{ccc} u_1''+(1+r)^2u_2'' & -\dfrac{(1+r)^2}{(1+np)^2}(G_1^A+G_2^A)u_2'' & u_1''+\dfrac{np(1+r)^2}{1+np}u_2'' \\ \dfrac{u_1''}{1+np} & -\dfrac{1}{(1+np)^2}u_1' & \dfrac{u_1''}{1+np} \\ u_1'' & \dfrac{G_2^A}{(1+np)^2}g_2'' & u_1''+\dfrac{g_2''}{1+np} \end{array}\right|$$

$$=\frac{1}{\Delta^A}\left\{-\frac{u_1'g_2''}{(1+np)^3}\left[u_1''+(1+r)^2u_2''\right]+\frac{(1+r)^2G_1^Au_1''u_2''g_2''}{(1+np)^4}-\frac{(1+r)^2u_1'u_1''u_2''}{(1+np)^3}\right\}>0,$$

$$\frac{dG_2^A}{d(np)}=\frac{1}{\Delta^A}\left|\begin{array}{ccc} u_1''+(1+r)^2u_2'' & u_1''+\dfrac{np(1+r)^2}{1+np}u_2'' & -\dfrac{(1+r)^2}{(1+np)^2}(G_1^A+G_2^A)u_2'' \\ \dfrac{u_1''}{1+np} & \dfrac{u_1''}{1+np}+g_1'' & -\dfrac{1}{(1+np)^2}u_1' \\ u_1'' & u_1'' & \dfrac{G_2^A}{(1+np)^2}g_2'' \end{array}\right|$$

$$=\frac{1}{\Delta^A}\left\{\frac{G_2^Ag_1''g_2''}{(1+np)^2}\left[u_1''+(1+r)^2u_2''\right]+\frac{(1+r)^2G_2^Au_1''u_2''g_2''}{(1+np)^4}+\frac{(1+r)^2u_1'u_1''u_2''}{(1+np)^3}+\frac{(1+r)^2(G_1^A+G_2^A)u_1''u_2''g_1''}{(1+np)^2}\right\},$$

$$\frac{d(G_1^A+G_2^A)}{d(np)}=\frac{1}{\Delta^A}\left\{\begin{array}{l} -\dfrac{u_1'g_2''}{(1+np)^3}\left[u_1''+(1+r)^2u_2''\right]+\dfrac{(1+r)^2G_1^Au_1''u_2''g_2''}{(1+np)^4}+\dfrac{G_2^Ag_1''g_2''}{(1+np)^2}\left[u_1''+(1+r)^2u_2''\right] \\ +\dfrac{(1+r)^2G_2^Au_1''u_2''g_2''}{(1+np)^4}+\dfrac{(1+r)^2(G_1^A+G_2^A)u_1''u_2''g_1''}{(1+np)^2} \end{array}\right\}>0.$$

Proof of Proposition 4

Differentiating the four equations in (20) with respect to p, respectively, yields

$$(G_1^B+G_2^B)\frac{d\alpha_B}{dp}+\alpha_B\left(\frac{dG_1^B}{dp}+\frac{dG_2^B}{dp}\right)=\left(\frac{1-p}{1+np}\right)\frac{d\left(G_1^A+G_2^A\right)}{dp}-\frac{1+n}{(1+np)^2}(G_1^A+G_2^A)$$

$$-\frac{(1+r)^2p(G_1^B+G_2^B)u_2''}{1-p}\frac{d\alpha_B}{dp}+\left[u_1''+(1+r)^2u_2''\right]\frac{dk^B}{dp}+\left[u_1''-\frac{(1+r)^2p\alpha_Bu_2''}{1-p}\right]\left(\frac{dG_1^B}{dp}+\frac{dG_2^B}{dp}\right)$$

$$=\frac{(1+r)^2\alpha_B(G_1^B+G_2^B)u_2''}{(1-p)^2}$$

$$u_1''\frac{dk^B}{dp}+\left(u_1''+g_1''\right)\frac{dG_1^B}{dp}+u_1''\frac{dG_2^B}{dp}=0$$

$$u_1''\frac{dk^B}{dp}+u_1''\frac{dG_1^B}{dp}+\left[u_1''+\left(\frac{1}{1-p}\right)g_2''\right]\frac{dG_2^B}{dp}=-\frac{G_2^B}{(1-p)^2}g_2''.$$

We have

$$\Delta^B\equiv\begin{vmatrix} G_1^B+G_2^B & 0 & \alpha_B & \alpha_B \\ -\frac{(1+r)^2p(G_1^B+G_2^B)u_2''}{1-p} & u_1''+(1+r)^2u_2'' & u_1''-\frac{(1+r)^2p\alpha_Bu_2''}{1-p} & u_1''-\frac{(1+r)^2p\alpha_Bu_2''}{1-p} \\ 0 & u_1'' & u_1''+g_1'' & u_1'' \\ 0 & u_1'' & u_1'' & u_1''+\left(\frac{1}{1-p}\right)g_2'' \end{vmatrix}$$

$$=\left(G_1^B+G_2^B\right)\begin{vmatrix} u_1''+(1+r)^2u_2'' & u_1''-\frac{(1+r)^2p\alpha_Bu_2''}{1-p} & u_1''-\frac{(1+r)^2p\alpha_Bu_2''}{1-p} \\ u_1'' & u_1''+g_1'' & u_1'' \\ u_1'' & u_1'' & u_1''+\left(\frac{1}{1-p}\right)g_2'' \end{vmatrix}$$

$$+\frac{(1+r)^2p(G_1^B+G_2^B)u_2''}{1-p}\begin{vmatrix} 0 & \alpha_B & \alpha_B \\ u_1'' & u_1''+g_1'' & u_1'' \\ u_1'' & u_1'' & u_1''+\left(\frac{1}{1-p}\right)g_2'' \end{vmatrix}$$

$$=\left(G_1^B+G_2^B\right)\left\{\left(\frac{1}{1-p}\right)\left[u_1''+(1+r)^2u_2''\right]g_1''g_2''+(1+r)^2u_1''u_2''\left[g_1''+\left(\frac{1}{1-p}\right)g_2''\right]\right\}<0.$$

(i) $\frac{d\alpha_B}{dp}=\frac{\Delta_{\alpha_B}}{\Delta^B}$, where

$$\Delta_{\alpha_B} \equiv \begin{vmatrix} \left(\frac{1-p}{1+np}\right)\frac{d\left(G_1^A+G_2^A\right)}{dp}-\frac{1+n}{(1+np)^2}(G_1^A+G_2^A) & 0 & \alpha_B & \alpha_B \\ \frac{(1+r)^2\alpha_B(G_1^B+G_2^B)u_2''}{(1-p)^2} & u_1''+(1+r)^2u_2'' & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} \\ 0 & u_1'' & u_1''+g_1'' & u_1'' \\ -\frac{G_2^B}{(1-p)^2}g_2'' & u_1'' & u_1'' & u_1''+\left(\frac{1}{1-p}\right)g_2'' \end{vmatrix}$$

$$=\left[\left(\frac{1-p}{1+np}\right)\frac{d\left(G_1^A+G_2^A\right)}{dp}-\frac{1+n}{(1+np)^2}(G_1^A+G_2^A)\right]\begin{vmatrix} u_1''+(1+r)^2u_2'' & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} \\ u_1'' & u_1''+g_1'' & u_1'' \\ u_1'' & u_1'' & u_1''+\left(\frac{1}{1-p}\right)g_2'' \end{vmatrix}$$

$$+\frac{(1+r)^2\alpha_B^2(G_1^B+G_2^B)u_1''u_2''}{(1-p)^2}\left[g_1''+\left(\frac{1}{1-p}\right)g_2''\right]+\frac{\alpha_B G_2^B}{\left(1-p\right)^2}\left[u_1''+(1+r)^2u_2''\right]g_1''g_2'',$$

where $\begin{vmatrix} u_1''+(1+r)^2u_2'' & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} \\ u_1'' & u_1''+g_1'' & u_1'' \\ u_1'' & u_1'' & u_1''+\left(\frac{1}{1-p}\right)g_2'' \end{vmatrix} < 0$ can be readily checked.

So a sufficient condition for $\frac{d\alpha_B}{dp} > 0$ is (21).

(ii) $\frac{dk^B}{dp} = \frac{\Delta_{k^B}}{\Delta^B}$, where

$$\Delta_{k^B} \equiv \begin{vmatrix} G_1^B + G_2^B & \left(\frac{1-p}{1+np}\right)\frac{d\left(G_1^A+G_2^A\right)}{dp} - \frac{1+n}{(1+np)^2}(G_1^A+G_2^A) & \alpha_B & \alpha_B \\ -\frac{(1+r)^2 p(G_1^B+G_2^B)u_2''}{1-p} & \frac{(1+r)^2\alpha_B(G_1^B+G_2^B)u_2''}{(1-p)^2} & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} \\ 0 & 0 & u_1''+g_1'' & u_1'' \\ 0 & -\frac{G_2^B}{(1-p)^2}g_2'' & u_1'' & u_1''+\left(\frac{1}{1-p}\right)g_2'' \end{vmatrix}$$

$$=\left(G_1^B+G_2^B\right)\begin{vmatrix} \frac{(1+r)^2\alpha_B(G_1^B+G_2^B)u_2''}{(1-p)^2} & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} \\ 0 & u_1''+g_1'' & u_1'' \\ -\frac{G_2^B}{(1-p)^2}g_2'' & u_1'' & u_1''+\left(\frac{1}{1-p}\right)g_2'' \end{vmatrix}$$

$$+\frac{(1+r)^2 p(G_1^B+G_2^B)u_2''}{1-p}\begin{vmatrix} \left(\frac{1-p}{1+np}\right)\frac{d\left(G_1^A+G_2^A\right)}{dp} - \frac{1+n}{(1+np)^2}(G_1^A+G_2^A) & \alpha_B & \alpha_B \\ 0 & u_1''+g_1'' & u_1'' \\ -\frac{G_2^B}{(1-p)^2}g_2'' & u_1'' & u_1''+\left(\frac{1}{1-p}\right)g_2'' \end{vmatrix}$$

$$=\left(G_1^B+G_2^B\right)\begin{vmatrix} \frac{(1+r)^2\alpha_B(G_1^B+G_2^B)u_2''}{(1-p)^2} & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} \\ 0 & u_1''+g_1'' & u_1'' \\ -\frac{G_2^B}{(1-p)^2}g_2'' & u_1'' & u_1''+\left(\frac{1}{1-p}\right)g_2'' \end{vmatrix}$$

$$+\frac{(1+r)^2 p(G_1^B+G_2^B)u_2''}{1-p}\left\{\left[\left(\frac{1-p}{1+np}\right)\frac{d\left(G_1^A+G_2^A\right)}{dp} - \frac{1+n}{(1+np)^2}(G_1^A+G_2^A)\right]\left[\left(\frac{1}{1-p}\right)u_1''g_2''+u_1''g_1''+\left(\frac{1}{1-p}\right)g_1''g_2''\right]+\frac{\alpha_B G_2^B}{(1-p)^2}g_1''g_2''\right\}$$

where $\begin{vmatrix} \frac{(1+r)^2\alpha_B(G_1^B+G_2^B)u_2''}{(1-p)^2} & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} \\ 0 & u_1''+g_1'' & u_1'' \\ -\frac{G_2^B}{(1-p)^2}g_2'' & u_1'' & u_1''+\left(\frac{1}{1-p}\right)g_2'' \end{vmatrix} < 0$ can be readily checked.

So a sufficient condition for $\frac{dk^B}{dp} > 0$ is (21).

(iii) $\frac{dG_2^B}{dp} = \frac{\Delta_{G_2^B}}{\Delta^B}$, where

$$\Delta_{G_2^B} \equiv \begin{vmatrix} G_1^B + G_2^B & 0 & \alpha_B & \left(\frac{1-p}{1+np}\right)\frac{d\left(G_1^A+G_2^A\right)}{dp} - \frac{1+n}{(1+np)^2}(G_1^A+G_2^A) \\ -\frac{(1+r)^2 p(G_1^B+G_2^B)u_2''}{1-p} & u_1''+(1+r)^2 u_2'' & u_1'' - \frac{(1+r)^2 p\alpha_B u_2''}{1-p} & \frac{(1+r)^2\alpha_B(G_1^B+G_2^B)u_2''}{(1-p)^2} \\ 0 & u_1'' & u_1''+g_1'' & 0 \\ 0 & u_1'' & u_1'' & -\frac{G_2^B}{(1-p)^2}g_2'' \end{vmatrix}$$

$$= \left(G_1^B+G_2^B\right)\begin{vmatrix} u_1''+(1+r)^2 u_2'' & u_1'' - \frac{(1+r)^2 p\alpha_B u_2''}{1-p} & \frac{(1+r)^2\alpha_B(G_1^B+G_2^B)u_2''}{(1-p)^2} \\ u_1'' & u_1''+g_1'' & 0 \\ u_1'' & u_1'' & -\frac{G_2^B}{(1-p)^2}g_2'' \end{vmatrix}$$

$$+\frac{(1+r)^2 p(G_1^B+G_2^B)u_2''}{1-p}\left\{-\frac{\alpha_B G_2^B}{(1-p)^2}u_1''g_2'' - u_1''g_1''\left[\left(\frac{1-p}{1+np}\right)\frac{d\left(G_1^A+G_2^A\right)}{dp} - \frac{1+n}{(1+np)^2}(G_1^A+G_2^A)\right]\right\},$$

where $\begin{vmatrix} u_1''+(1+r)^2 u_2'' & u_1'' - \frac{(1+r)^2 p\alpha_B u_2''}{1-p} & \frac{(1+r)^2\alpha_B(G_1^B+G_2^B)u_2''}{(1-p)^2} \\ u_1'' & u_1''+g_1'' & 0 \\ u_1'' & u_1'' & -\frac{G_2^B}{(1-p)^2}g_2'' \end{vmatrix} > 0$ can be readily checked.

So a sufficient condition for $\frac{dG_2^B}{dp} < 0$ is (21)

(iv) From the first equation in (20), it immediately follows that $\frac{d\left[\alpha_B(G_1^B+G_2^B)\right]}{dp} \geq 0$ if and only if (21) holds.

Proof of Proposition 5

Differentiating the four equations in (20) with respect to n, respectively, yields

$$(G_1^B+G_2^B)\frac{d\alpha_B}{dn}+\alpha_B\left(\frac{dG_1^B}{dn}+\frac{dG_2^B}{dn}\right)=\left(\frac{1-p}{1+np}\right)\frac{d\left(G_1^A+G_2^A\right)}{dn}-\frac{p(1-p)}{(1+np)^2}(G_1^A+G_2^A)$$

$$-\frac{(1+r)^2 p(G_1^B+G_2^B)u_2''}{1-p}\frac{d\alpha_B}{dn}+\left[u_1''+(1+r)^2u_2''\right]\frac{dk^B}{dn}+\left[u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p}\right]\left(\frac{dG_1^B}{dn}+\frac{dG_2^B}{dn}\right)=0$$

$$u_1''\frac{dk^B}{dn}+\left(u_1''+g_1''\right)\frac{dG_1^B}{dn}+u_1''\frac{dG_2^B}{dn}=0$$

$$u_1''\frac{dk^B}{dn}+u_1''\frac{dG_1^B}{dn}+\left[u_1''+\left(\frac{1}{1-p}\right)g_2''\right]\frac{dG_2^B}{dn}=0.$$

We have

$$\Delta^B\equiv\begin{vmatrix} G_1^B+G_2^B & 0 & \alpha_B & \alpha_B \\ -\frac{(1+r)^2 p(G_1^B+G_2^B)u_2''}{1-p} & u_1''+(1+r)^2u_2'' & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} \\ 0 & u_1'' & u_1''+g_1'' & u_1'' \\ 0 & u_1'' & u_1'' & u_1''+\left(\frac{1}{1-p}\right)g_2'' \end{vmatrix}<0,$$

from the proof of Proposition 4.

We thus have

$$\frac{d\alpha_B}{dn}=\frac{1}{\Delta^B}\begin{vmatrix} \left(\frac{1-p}{1+np}\right)\frac{d\left(G_1^A+G_2^A\right)}{dn}-\frac{p(1-p)}{(1+np)^2}(G_1^A+G_2^A) & 0 & \alpha_B & \alpha_B \\ 0 & u_1''+(1+r)^2u_2'' & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} \\ 0 & u_1'' & u_1''+g_1'' & u_1'' \\ 0 & u_1'' & u_1'' & u_1''+\left(\frac{1}{1-p}\right)g_2'' \end{vmatrix}$$

$$=\frac{1}{\Delta^B}\left[\left(\frac{1-p}{1+np}\right)\frac{d\left(G_1^A+G_2^A\right)}{dn}-\frac{p(1-p)}{(1+np)^2}(G_1^A+G_2^A)\right]\begin{vmatrix} u_1''+(1+r)^2u_2'' & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} & u_1''-\frac{(1+r)^2 p\alpha_B u_2''}{1-p} \\ u_1'' & u_1''+g_1'' & u_1'' \\ u_1'' & u_1'' & u_1''+\left(\frac{1}{1-p}\right)g_2'' \end{vmatrix}$$

$$\text{where} \begin{vmatrix} u_1'' + (1+r)^2 u_2'' & u_1'' - \dfrac{(1+r)^2 p\alpha_B u_2''}{1-p} & u_1'' - \dfrac{(1+r)^2 p\alpha_B u_2''}{1-p} \\ u_1'' & u_1'' + g_1'' & u_1'' \\ u_1'' & u_1'' & u_1'' + \left(\dfrac{1}{1-p}\right) g_2'' \end{vmatrix} < 0$$

from the proof of Proposition 4.

So $\frac{d\alpha_B}{dn} > 0$ if and only if (22) holds. It can be similarly shown that (22) is also the sufficient and necessary condition for $\frac{dk^B}{dn} > 0, \frac{dG_1^B}{dn} < 0, \frac{dG_2^B}{dn} < 0,$ and $\frac{d\left[\alpha_B (G_1^B + G_2^B)\right]}{dn} > 0$.

Proof of Proposition 1'

Differentiating the three equations in (4') with respect to p, respectively, taking k^*, G_1^*, G_2^* as functions of p, yields

$$\left[(1+r)^2 v_{cc} + u_{cc}\right]\frac{dk^*}{dp} + (u_{cc} - u_{cG})\frac{dG_1^*}{dp} + \left[(1+r)v_{cG} + u_{cc}\right]\frac{dG_2^*}{dp} = 0$$

$$\left[(1-p)u_{cc} - u_{cG}\right]\frac{dk^*}{dp} + \left[(1-p)(u_{cc} - u_{cG}) - u_{cG} + u_{GG}\right]\frac{dG_1^*}{dp} + \left[(1-p)u_{cc} - u_{cG}\right]\frac{dG_2^*}{dp} = -u_1'.$$

$$\left[u_{cc} + (1+r)v_{cG}\right]\frac{dk^*}{dp} + (u_{cc} - u_{cG})\frac{dG_1^*}{dp} + (u_{cc} + v_{GG})\frac{dG_2^*}{dp} = 0,$$

in which we have used the fact that $u_{cG} = u_{Gc}$ and $v_{cG} = v_{Gc}$ based on Young's Theorem. Let

$$\begin{aligned}\Delta \equiv &\begin{vmatrix} (1+r)^2 v_{cc} + u_{cc} & u_{cc} - u_{cG} & (1+r)v_{cG} + u_{cc} \\ (1-p)u_{cc} - u_{cG} & (1-p)(u_{cc} - u_{cG}) - u_{cG} + u_{GG} & (1-p)u_{cc} - u_{cG} \\ u_{cc} + (1+r)v_{cG} & u_{cc} - u_{cG} & u_{cc} + v_{GG} \end{vmatrix} \\ &= \left[u_{cc}u_{GG} - (u_{cG})^2\right]\left\{(1+r)\left[(1+r)v_{cc} - v_{cG}\right] + v_{GG} - (1+r)v_{cG}\right\} \\ &\quad + \left[v_{cc}v_{GG} - (v_{cG})^2\right](1+r)^2\left[(1-p)u_{cc} + (p-2)u_{cG} + u_{GG}\right] < 0.\end{aligned}$$

Therefore,

$$\frac{dk^*}{dp} = \frac{1}{\Delta}\begin{vmatrix} 0 & u_{cc} - u_{cG} & (1+r)v_{cG} + u_{cc} \\ -u_c & (1-p)(u_{cc} - u_{cG}) - u_{cG} + u_{GG} & (1-p)u_{cc} - u_{cG} \\ 0 & u_{cc} - u_{cG} & u_{cc} + v_{GG} \end{vmatrix}$$

$$= \frac{u_c}{\Delta}(u_{cc} - u_{cG})\left[v_{GG} - (1+r)v_{cG}\right] < 0,$$

$$\frac{dG_1^*}{dp} = \frac{1}{\Delta}\begin{vmatrix} (1+r)^2 v_{cc} + u_{cc} & 0 & (1+r)v_{cG} + u_{cc} \\ (1-p)u_{cc} - u_{cG} & -u_c & (1-p)u_{cc} - u_{cG} \\ u_{cc} + (1+r)v_{cG} & 0 & u_{cc} + v_{GG} \end{vmatrix}$$

$$= \frac{-u_c}{\Delta}\left\{\left[(1+r)^2 v_{cc} + v_{GG} - 2(1+r)v_{cG}\right]u_{cc} + (1+r)^2\left[v_{cc}v_{GG} - (v_{cG})^2\right]\right\} > 0,$$

$$\frac{dG_2^*}{dp} = \frac{1}{\Delta}\begin{vmatrix} (1+r)^2 v_{cc} + u_{cc} & u_{cc} - u_{cG} & 0 \\ (1-p)u_{cc} - u_{cG} & (1-p)(u_{cc} - u_{cG}) - u_{cG} + u_{GG} & -u_c \\ u_{cc} + (1+r)v_{cG} & u_{cc} - u_{cG} & 0 \end{vmatrix}$$

$$= \frac{u_c}{\Delta}(u_{cc} - u_{cG})(1+r)\left[(1+r)v_{cc} - v_{cG}\right] < 0.$$

Moreover,

$$\frac{d(G_1^* + G_2^*)}{dp} = \frac{u_c}{\Delta}\left\{u_{cc}\left[(1+r)v_{cG} - v_{GG}\right] + (1+r)u_{cG}\left[v_{cG} - (1+r)v_{cc}\right] + (1+r)^2\left[(v_{cG})^2 - v_{cc}v_{GG}\right]\right\} > 0$$

if $u_{cc}\left[(1+r)v_{cG} - v_{GG}\right] < (1+r)u_{cG}\left[(1+r)v_{cc} - v_{cG}\right]$.