

# CAVITY QUANTUM ELECTRODYNAMICS WITH LANDAU QUANTIZED GRAPHENE

An Undergraduate Research Scholars Thesis

by

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# ABSTRACT

Cavity Quantum Electrodynamics with Landau Quantized Graphene

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We attempt to see if strong coupling between a quantum electromagnetic field and Landau quantized (LQ) graphene is achievable. LQ graphene acts as a collection of two-level quantum bits (qubits). This coupling between field and qubits gives rise to Rabi oscillations between the two energy levels of the qubits. The cavity we are using to confine the optical field is a photonic crystal cavity with a 2D sheet of graphene sitting between two layers of hexagonal boron nitride (hBN) with distributed Bragg reflectors (DBRs) on the outside acting as mirrors.

A better understanding would make it possible for devices to be constructed that implement new principles of telecommunications and computing. For example, quantum computers utilize qubits constructed of the superposition of quantum states instead of classical bits based on electric charge. Quantum computing promises drastic increase in memory density and computation speed. Communications based on the exchange of quantum states of light are inherently secure, because any eavesdropping attempt will destroy the quantum state.

## **DEDICATION**

*To Julia Tiller, who kept me company as I wrote this paper while she wrote her own.*

## **ACKNOWLEDGMENTS**

### **Contributors**

I would like to thank my faculty advisor, Dr. Belyanin, for his guidance and support throughout the course of this research as well as my collaborators Karthik Rao and Jacob Woods for their input during this research project.

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All other work conducted for the thesis was completed by the student independently.

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## NOMENCLATURE

$ \Psi\rangle$	The state vector representing a quantum system
$\hat{H}$	The Hamiltonian operator, the generator of time translations
$\mathbf{E}$	The electric field vector
$\hat{\mathbf{E}}$	The quantized electric field operator
$c, c^\dagger$	The annihilation and creation operators for the quantized electric field, respectively
$\hat{\sigma}_j, \hat{\sigma}_j^\dagger$	The dipole operators, respectively
$\hat{p}_i = -i\hbar \frac{\partial}{\partial x_i}$	The canonical momentum operator
$\hat{\Pi}_i = \hat{p}_i + \frac{e}{c} A_i$	The mechanical momentum operator
$i\hbar \frac{\partial  \Psi\rangle}{\partial t} = \hat{H} \Psi\rangle$	The time-dependent Schrodinger equation
$\hat{H} \Psi\rangle = E \Psi\rangle$	The time-independent Schrodinger equation

# 1. INTRODUCTION

We want to mathematically describe light's interaction with graphene in a photonic crystal cavity within the rigorous quantum formalism. For every model with increasing complexity, we must write down a Schrodinger equation and solve it for the time evolution of the system. This is well-known but will become more difficult when we include coupling of a quantum system to an external world and associated dissipation and noise, as in our model in [1]. Several ways of doing this are explained in [2], such as Heisenberg-Langevin approach to adding damping and noise to your equations. Noise and damping are difficult barriers to overcome physically because these reduce the effectiveness of the coupling, hence the need for strong coupling between the electromagnetic field of the light and the atoms in the material as is described in the [3].

Many of the applications of strongly coupled nanophotonic systems are discussed in [4]. For example, better waveguides for light can be created that are highly tunable. The photons of incident light and the plasma of electrons in the material (such as a metal) become coupled and can be guided through nanowires like current in the form of plasmon-polaritons. This gives rise to applications in integrated photonics for optical computing and interconnects between computer parts. Quantum light leads to quantum entanglement between individual plasmons and electron states. Even though the energy of the electrons themselves can be difficult to change, the energy of the coupled states is easy to change by varying the frequency of the incident light or the geometry of the optical cavity [4].

In this paper we will use the standard convention that bolded characters such as  $\mathbf{E}$  and  $\mathbf{p}$  stand for vector valued quantities and characters with a carrot or "hat" on top such as  $\hat{\mathbf{E}}$  and  $\hat{\sigma}$  are linear operators unless otherwise stated.



## 2. GRAPHENE

We begin with an examination of the properties of graphene. Graphene is a two-dimensional hexagonal lattice of carbon atoms.

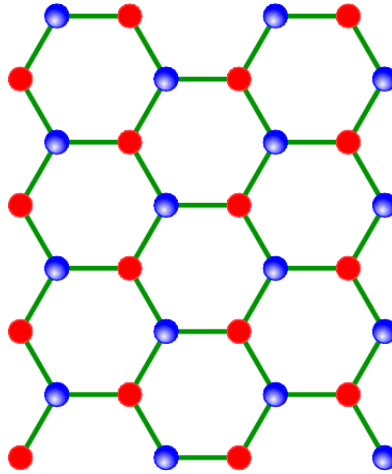


Figure 2.1: The red and blue atoms demonstrate the two triangular sublattices that compose the graphene sheet [5].

Carbon atoms have four valence electrons. Three of these form  $\sigma$  bonds (not to be confused with the dipole operators that will be introduced later which, regrettably, are also denoted  $\hat{\sigma}$ ) with the neighboring carbon atoms to form the hexagonal lattice like in Fig. 2.1. The fourth valence electron is the conduction electron that can move through the graphene.

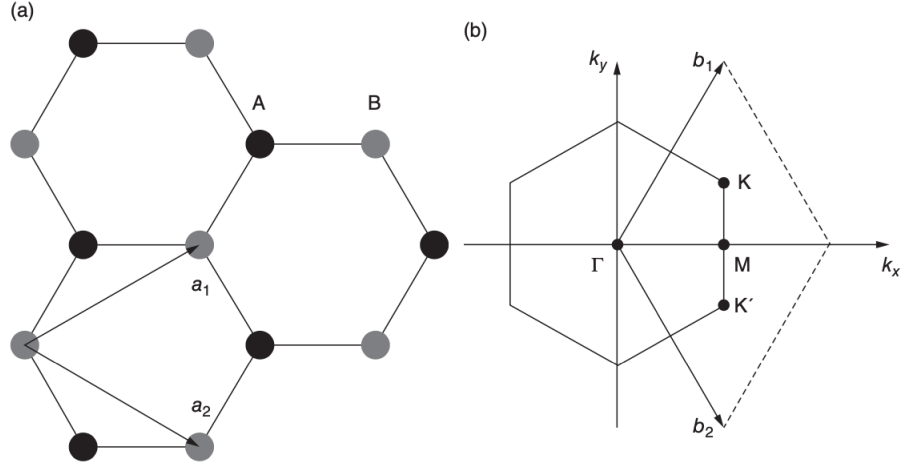


Figure 2.2: (a) shows the lattice structure of graphene and (b) shows the structure of the lattice in inverse space [6].

One can define nearest neighbor vectors  $\delta_{1,2,3}$  as well as the inverse lattice vectors for graphene  $\mathbf{b}_{1,2}$  given in [6]:

$$\delta_{1,2} = \frac{a}{2}(1, \pm\sqrt{3}), \quad \delta_3 = a(-1, 0) \quad (\text{Eq. 1})$$

and

$$\mathbf{b}_{1,2} = \frac{2\pi}{3a}(1, \pm\sqrt{3}). \quad (\text{Eq. 2})$$

One can think of the inverse lattice as taking place in momentum space (up to units of  $\hbar$ ), the Fourier transform of real space. The inverse space has points of high symmetry that we call  $K$  and  $K'$  given in [6] by

$$\mathbf{K}' = \frac{2\pi}{3a}\left(1, \frac{1}{\sqrt{3}}\right), \quad \mathbf{K} = \frac{2\pi}{3a}\left(1, \frac{1}{\sqrt{3}}\right). \quad (\text{Eq. 3})$$

Fig. 2.2 shows the real space lattice, inverse lattice, and  $K$  and  $K'$  points. The points  $K$  and  $K'$  are called *Dirac points*.

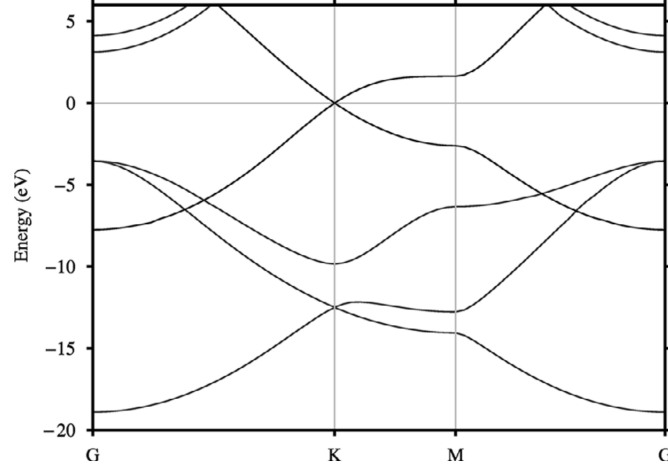


Figure 2.3: This figure shows the band structure of graphene. The band crossing at  $K$  comes from these “free” electrons. The three electrons that take place in the  $\sigma$  bonds form the bands that do not have these self-intersections [6].

In this paper, we focus on the band structure around these Dirac points. Around these points the energy as a function of momentum is approximately linear. This can be seen at the intersections of the bands at  $K$  in Fig. 2.3. The approximate Hamiltonian for the electrons around the  $K$  point is given in [7] and is

$$\hat{H} = v_F \hat{\boldsymbol{\sigma}} \cdot \hat{\mathbf{p}} = v_F \begin{pmatrix} 0 & \hat{p}_x - i\hat{p}_y \\ \hat{p}_x + i\hat{p}_y & 0 \end{pmatrix}, \quad (\text{Eq. 4})$$

where  $v_F = 10^6$  m/s is the Fermi velocity,  $\hat{p}_i = -i\hbar \frac{\partial}{\partial x_i}$  is the momentum operator, and  $\hat{\boldsymbol{\sigma}} = \hat{\sigma}_x \hat{x} + \hat{\sigma}_y \hat{y}$  is the vector composed of the three Pauli matrices (again not to be confused with the dipole moment operators). Eq. 4 and  $\hat{\boldsymbol{\sigma}}$  don’t include a  $z$ -component because the electrons are constrained to move on the two dimensional surface and we choose coordinates such that the  $z$ -axis is perpendicular to the sheet of graphene.

### 3. LANDAU QUANTIZED GRAPHENE

We now apply a constant magnetic field  $\mathbf{B} = B\hat{z}$  along the  $z$ -axis perpendicular to the sheet of graphene. To change our equations of motion, we make the replacement in the Hamiltonian in Eq. 4

$$\hat{\mathbf{p}} \mapsto \hat{\boldsymbol{\Pi}} = \hat{\mathbf{p}} + \frac{e}{c}\mathbf{A}, \quad (\text{Eq. 1})$$

where  $\mathbf{A}$  is the magnetic vector potential, implicitly defined by the equation  $\nabla \times \mathbf{A} = \mathbf{B}$  [7]. We further follow the convention of [7] and choose the gauge such that  $\mathbf{A} = (0, Bx, 0)$ . Then  $\hat{\Pi}_x = \hat{p}_x = -i\hbar\frac{\partial}{\partial x}$  and  $\hat{\Pi}_y = \hat{p}_y + \frac{e}{c}Bx = -i\hbar\frac{\partial}{\partial y} + \frac{e}{c}Bx$ .

One can solve for the energy eigenvalues  $E$  and eigenspinors  $\psi = (\psi_a, \psi_b)^T$  of this system by using the time-independent Schrodinger equation  $\hat{H}\psi = E\psi$ , or

$$\begin{pmatrix} 0 & \hat{p}_x - i(\hat{p}_y + \frac{e}{c}Bx) \\ \hat{p}_x + i(\hat{p}_y + \frac{e}{c}Bx) & 0 \end{pmatrix} \begin{pmatrix} \psi_a \\ \psi_b \end{pmatrix} = E \begin{pmatrix} \psi_a \\ \psi_b \end{pmatrix}. \quad (\text{Eq. 2})$$

The solution to this system of equations gives  $E = \pm\hbar\omega_c\sqrt{|n|}$  for  $n \in \mathbf{Z}$ , where  $\omega_c = \frac{\sqrt{2}v_F}{l_c}$  is the cyclotron frequency and  $l_c = \sqrt{\frac{c\hbar}{eB}}$  is the magnetic length [7].

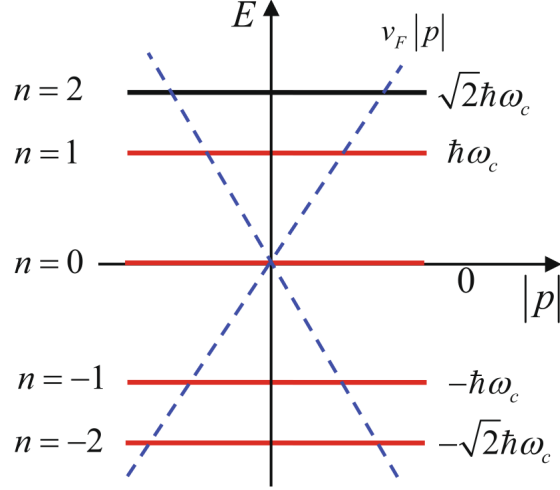


Figure 3.1: The cone represents the dispersion of the electrons in graphene near a Dirac point of the graphene. The horizontal Landau levels represent the possible energy levels for the electrons once the magnetic field  $\mathbf{B}$  is applied [8].

We will place the Fermi-level of our system  $E_F$  between the  $n = 0$  and  $n = 1$  states of our system, see Fig. 3.1. This means that all of the energy levels less than  $E_F$  (that is,  $n = 0$  and below) are fully occupied and the states  $n = 1$  and above are unoccupied.

When we introduce an optical electromagnetic field with photons that have an energy of approximately  $\hbar\omega_c$ , the interaction will excite some of the  $n = 0$  states to  $n = 1$  states. These transitions between  $n = 0$  and  $n = 1$  are our qubits. The number of qubits is given by the degeneracy of the 0th Landau level. For  $n > 0$  the degeneracy is given by the equation  $N = gA/(2\pi l_c^2)$ , where  $g = 2 \cdot 2 = 4$  is the spin-valley degeneracy and  $A$  is the area of the sheet of graphene [9]. For  $n = 0$ , the degeneracy is half this number [10]. Thus, the number of qubits is

$$N = \frac{A}{\pi l_c^2} = \frac{AeB}{\pi c\hbar}. \quad (\text{Eq. 3})$$

Note that the number of qubits is proportional to the magnetic field.

## 4. QUBITS IN AN OPTICAL FIELD

We will move on to an abstract discussion of a single qubit in an electromagnetic field. In this section, we will often refer to our qubit as a two-state atom, but it is ultimately the qubits in the Landau quantized graphene.

We are working with a quantized optical field so that the vector  $\mathbf{E}$  is promoted to a vector operator  $\hat{\mathbf{E}} = \mathbf{E}\hat{c} + \mathbf{E}^*\hat{c}^\dagger$ . The operators  $\hat{c}$  and  $\hat{c}^\dagger$  are the respective annihilation and creation operators for modes of the electric field, i.e., photons. In general, you would have separate creation and annihilation operators for each mode (or operators that are functions of the frequency depending on how you like to think about it) but as we have one mode, we just have the one set of operators. In this case these operators create modes of frequency  $\omega$  since we have just one field. These operators act on the Fock space of photons in the following manner:

$$c|n\rangle = \sqrt{n}|n-1\rangle, c^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle. \quad (\text{Eq. 1})$$

This normalization is often chosen so that the operator  $c^\dagger c$  acts on the state  $|n\rangle$  to give  $n|n\rangle$ . For this reason,  $c^\dagger c$  is often called the number operator. Since photons are bosons, the creation and annihilation operators obey the commutation relation

$$[c, c^\dagger] := cc^\dagger - c^\dagger c = 1, \quad (\text{Eq. 2})$$

where 1 is the identity operator on the Fock space.

The state vector for the system  $|\Psi\rangle$  can be expanded in basis states

$$|\Psi\rangle = C_{00}|0\rangle \prod_{j=1}^N |0_j\rangle + C_{10}|1\rangle \prod_{j=1}^N |0_j\rangle + \sum_{j=1}^N C_{0j}|0\rangle|1_j\rangle \prod_{m \neq j}^N |0_m\rangle. \quad (\text{Eq. 3})$$

Here the leftmost ket in each term represents the Fock states of the optical field. We only consider states of zero or one photons. The product  $\prod_{j=1}^N |0_j\rangle$  is the state of all  $N$  qubits being in the

unexcited state and  $|1_j\rangle \prod_{m \neq j}^N |0_m\rangle$  is the state where the  $j$ th qubit is excited and the rest are unexcited.

Since the field is a quantized object, we must include a term in the (quantum) Hamiltonian for the electric field. As shown in [2] this can be written,

$$H_{em} = \hbar\omega \left( c^\dagger c + \frac{1}{2} \right). \quad (\text{Eq. 4})$$

This makes sense intuitively because  $c^\dagger c$  represents the number of photons making up the field and each photon has an energy of  $\hbar\omega$ .

We need a term in the Hamiltonian for the qubit itself. That is, the valence electron can either be in a ground state  $|0\rangle$  or an excited state  $|1\rangle$ . Since the zero of the energy is arbitrary, we might as well set the ground state to be the zero energy state of the electron and let the excited state have energy  $W$ . This  $W$  is the energy difference  $\hbar\omega_c$  in the case of our Landau quantized graphene. The "free" Hamiltonian for the qubits without the field interaction is then simply

$$\hat{H}_a = \sum_{j=1}^N W |1_j\rangle \langle 1_j|, \quad (\text{Eq. 5})$$

where the subscript  $a$  is for "atom" and the sum on  $j$  is for the  $N$  qubits [11].

We must also write the interaction Hamiltonian in terms of the field operator. We have the dipole moment operator  $\hat{\mathbf{d}} = \sum_{j=1}^N \mathbf{d} |1_j\rangle \langle 0_j| + \mathbf{d}^* |0_j\rangle \langle 1_j|$ . If we define the operator  $\hat{\sigma}_j := |0_j\rangle \langle 1_j|$  then the dipole moment operator can be written

$$\hat{\mathbf{d}} = \sum_{j=1}^N \mathbf{d} \hat{\sigma}_j^\dagger + \mathbf{d}^* \hat{\sigma}_j. \quad (\text{Eq. 6})$$

It is worthwhile for calculations to note some properties of the operators  $\hat{\sigma}$  and  $\hat{\sigma}^\dagger$ . We list them below:

- $\hat{\sigma}_j^\dagger |0_j\rangle = |1_j\rangle$ ,  $\hat{\sigma}_j^\dagger |1_j\rangle = 0$

- $\hat{\sigma}_j|1_j\rangle = |0_j\rangle, \hat{\sigma}_j|0_j\rangle = 0$
- $\hat{\sigma}_j^2 = (\hat{\sigma}_j^\dagger)^2 = 0$
- $\{\hat{\sigma}_j, \hat{\sigma}_j^\dagger\} := \hat{\sigma}_j\hat{\sigma}_j^\dagger + \hat{\sigma}_j^\dagger\hat{\sigma}_j = 1.$

Note that  $\hat{\sigma}_j$  and  $\hat{\sigma}_j^\dagger$  obey an anti-commutation relation instead of a commutation relation like the photon field operators. This allows us to write the atomic Hamiltonian in the form

$$\hat{H}_a = \sum_{j=1}^N W \hat{\sigma}_j^\dagger \hat{\sigma}_j. \quad (\text{Eq. 7})$$

Then the interaction Hamiltonian is simply  $\hat{H}_1 = -\hat{\mathbf{d}} \cdot \hat{\mathbf{E}}$  so that as in [11] the total Hamiltonian for the system is

$$\hat{H} = \hat{H}_a + \hat{H}_{em} + \hat{H}_1 = \hbar\omega \left( c^\dagger c + \frac{1}{2} \right) + \sum_{j=1}^N \left[ W \hat{\sigma}_j^\dagger \hat{\sigma}_j - (\mathbf{d} \hat{\sigma}_j^\dagger + \mathbf{d}^* \hat{\sigma}_j) \cdot (\mathbf{E} \hat{c} + \mathbf{E}^* \hat{c}^\dagger) \right]. \quad (\text{Eq. 8})$$

We will now also employ the *rotating wave approximation*. That is, we assume the Rabi frequency  $\Omega_R := \mathbf{d} \cdot \mathbf{E}/\hbar$  as well as the difference  $|W - \hbar\omega|$  are both  $\ll \hbar\omega, W$ . This allows us to ignore the terms that will show up in the equations as  $e^{\pm i(\omega+W/\hbar)t}$  because they rotate with such a high frequency that they average to zero over any significant amount of time. In practice, this amounts to throwing away the terms in the Hamiltonian that include  $\mathbf{d}^* \cdot \mathbf{E}$  and  $\mathbf{d} \cdot \mathbf{E}^*$  so we have

$$\hat{H} = \hbar\omega \left( c^\dagger c + \frac{1}{2} \right) + \sum_{j=1}^N \left[ W \hat{\sigma}_j^\dagger \hat{\sigma}_j - \hbar\Omega_R \hat{\sigma}_j^\dagger \hat{c} - \hbar\Omega_R^* \hat{\sigma}_j \hat{c}^\dagger \right]. \quad (\text{Eq. 9})$$

We substitute  $|\Psi\rangle$  from Eq. 3 into the Schrodinger equation  $i\hbar \frac{\partial |\Psi\rangle}{\partial t} = \hat{H}|\Psi\rangle$  using the Hamiltonian



Eq. 9 to obtain the equations

$$\dot{C}_{00} = \frac{1}{i} \frac{\omega}{2} C_{00} \quad (\text{Eq. 10})$$

$$\dot{C}_{10} = \frac{1}{i} \left( \frac{3}{2} \omega C_{10} - \sum_{j=1}^N \Omega_R^* C_{0j} \right) \quad (\text{Eq. 11})$$

$$\dot{C}_{0j} = \frac{1}{i} \left( \frac{\omega}{2} C_{0j} - \Omega_R C_{10} + \frac{W}{\hbar} C_{0j} \right), \quad (\text{Eq. 12})$$

where the dot over a letter such as  $\dot{C}$  indicates a time derivative. The solution for  $C_{00}$  is simply  $C_{00}(t) = C_{00}(0)e^{-i\omega t/2}$ . Following the convention in [11], we define  $F := \sum_{j=1}^N C_{0j}$  so that we can obtain the equations

$$\dot{C}_{10} = \frac{1}{i} \left( \frac{3}{2} \omega C_{10} - \Omega_R^* F \right) \quad (\text{Eq. 13})$$

$$\dot{F} = -i \left( \frac{\omega}{2} F - N \Omega_R C_{10} + \frac{W}{\hbar} F \right), \quad (\text{Eq. 14})$$

We now have coupled equations for  $C_{10}$  and  $F$  that are more straightforward to solve. One further simplification we can make is by factoring out a phase  $e^{-i3\omega t/2}$  by defining  $C_{10} = G_{10}e^{-i3\omega t/2}$  and  $F = G_F e^{-i3\omega t/2}$  [11]. After defining  $\Delta := \frac{W}{\hbar} - \omega$  and doing some slight rearranging we arrive at the equation

$$\ddot{G}_{10} + i\Delta \dot{G}_{10} + N|\Omega_R|^2 G_{10} = 0 \quad (\text{Eq. 15})$$

which has solution

$$G_{10}(t) = A_1 e^{\Gamma_1 t} + A_2 e^{\Gamma_2 t}, \quad (\text{Eq. 16})$$

where

$$\Gamma_{1,2} = -i \left( \frac{\Delta}{2} \pm \sqrt{\frac{\Delta^2}{4} + N|\Omega_R|^2} \right). \quad (\text{Eq. 17})$$

Finally, this gives for  $C_{0j}$  [11]

$$C_{0j}(t) = i\Omega_R e^{-i(\frac{3\omega}{2} + \Delta)t} \int_0^t e^{i\Delta\tau} G_{10}(\tau) d\tau. \quad (\text{Eq. 18})$$

## 5. THE PHOTONIC CRYSTAL CAVITY AND PUTTING IT ALL TOGETHER

### 5.1 The cavity

The cavity we study to confine our electromagnetic field is a photonic crystal cavity containing our Landau quantized graphene.

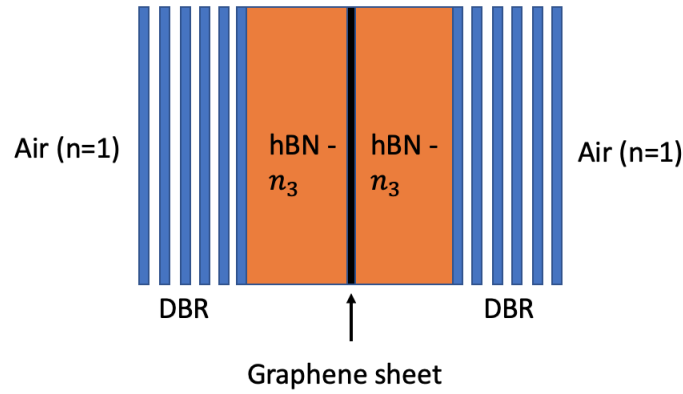


Figure 5.1: The photonic crystal cavity is designed using two DBR (distributed Bragg reflector) mirrors to confine the electric field to the hexagonal Boron Nitride (hBN) cavity. In the middle of the cavity is a 2D Landau quantized graphene sheet which contains the qubits we are studying.

The mirrors used to contain the optical field are distributed Bragg reflectors (DBRs). DBRs are stacks of a  $M$  pairs of alternating dielectrics with dielectric constant  $n_1$  and  $n_2$  (Fig. 5.1). These alternating stacks lead to constructive interference so that the stacks act as a mirror, confining the electric field to the cavity. The reflectivity of a DBR with  $M$  pairs is given by the well known relation [12]

$$R = \left( \frac{1 - \frac{n_s}{n_0} \left( \frac{n_1}{n_2} \right)^{2M}}{1 + \frac{n_s}{n_0} \left( \frac{n_1}{n_2} \right)^{2M}} \right)^2, \quad (\text{Eq. 1})$$

where  $n_0 = 1$  is the index of refraction of air,  $n_s$  is the index of refraction of the hBN, and  $n_1$  and  $n_2$  are the indices of refraction for the alternating dielectrics that make up the DBR.

## 5.2 Doing some calculations

Now let us attempt to do some rough calculations for our system. Since we are working in the rotating wave approximation, for the purpose of obtaining numbers for calculations we will take the frequency of the optical field to be  $\omega = \omega_c$ . Furthermore, we take the field to be normalized to be  $E \approx \sqrt{\frac{2\pi\hbar\omega}{V\varepsilon}}$ . The length, width, and height of the cavity are  $\frac{\lambda}{2n}$  where  $\lambda = \frac{2\pi c}{\omega}$  is the vacuum wavelength of the optical field.

The relative permittivity  $\varepsilon$  for the hBN is approximately 10. Then  $n_s = \sqrt{\varepsilon} \approx 3.16$ . The magnitude of the dipole moment is  $|d| = \frac{ev_F}{\omega}$  [7]. Recall that the Fermi-velocity is  $v_F = 10^6$  m/s. Take the magnetic field to be  $B = 8$  tesla, a magnetic field that can be easily produced in a lab. Then (in SI units)  $l_c = \sqrt{\frac{\hbar}{eB}} \approx 9.07 \cdot 10^{-9}$  m and  $\omega_c = \frac{\sqrt{2}v_F}{l_c} \approx 1.56 \cdot 10^{14}$  s<sup>-1</sup>. The vacuum wavelength is then  $\lambda = \frac{2\pi c}{\omega} \approx 1.21 \cdot 10^{-5}$  m. This gives a cavity area and volume of  $A = \left(\frac{\lambda}{2n}\right)^2 \approx 3.65 \cdot 10^{-12}$  m<sup>2</sup> and  $V = \left(\frac{\lambda}{2n}\right)^3 \approx 6.99 \cdot 10^{-18}$  m<sup>3</sup>. The number of qubits can now be calculated to be  $N = \frac{A}{\pi l_c^2} \approx 1.41 \cdot 10^4$ . The Rabi frequency in SI units is

$$\Omega_R \sim \frac{Ed}{\hbar} = \sqrt{\frac{2\pi\hbar\omega}{V\varepsilon\varepsilon_0}} \frac{ev_F}{\hbar\omega} = ev_F \sqrt{\frac{2\pi}{V\hbar\omega\varepsilon\varepsilon_0}} \approx 1.26 \cdot 10^{11} \text{ s}^{-1}. \quad (\text{Eq. 1})$$

We now plot the eigenenergies from Chapter 4,

$$\omega_{eigen} = -i\Gamma_{1,2} = -\frac{\Delta}{2} \pm \sqrt{\frac{\Delta^2}{4} + N|\Omega_R|^2}, \quad (\text{Eq. 2})$$

with  $\Delta = \omega_c - \omega$  as a function of the optical frequency  $\omega$  and using our values for  $N$  and  $\Omega_R$ . In this case SI units are slightly unwieldy so we will use meV. In these units  $\Omega_R = 0.0829$  meV and  $\omega_c = 103.$  meV.

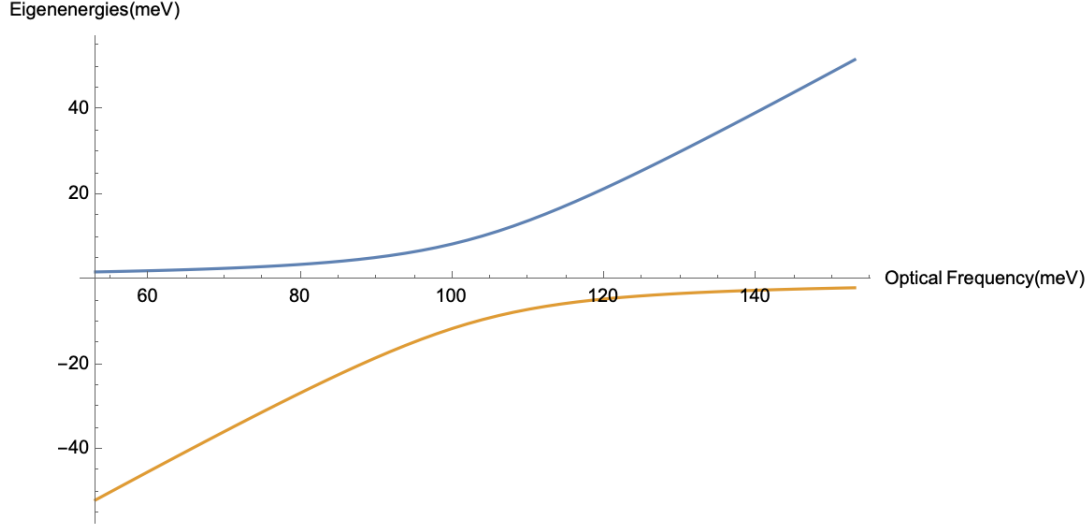


Figure 5.2: A plot of the anti-crossing signaling strong coupling between the optical field and the Landau quantized graphene.

For simplicity, let us take the case where  $\Delta = 0$ . That is,  $\omega = \omega_c$  exactly. In Fig. 5.2 this corresponds to the point where there is the smallest gap between the bands. Moreover, take the initial state to be  $|1\rangle \prod_{j=1}^N |0_j\rangle$  - the state with one photon excitation of the electromagnetic field and no excited qubits. With state vector

$$|\Psi(0)\rangle = C_{00}(0)|0\rangle \prod_{j=1}^N |0_j\rangle + C_{10}(0)|1\rangle \prod_{j=1}^N |0_j\rangle + \sum_{j=1}^N C_{0j}(0)|0\rangle |1_j\rangle \prod_{m \neq j}^N |0_m\rangle. \quad (\text{Eq. 3})$$

we can see that  $C_{00}(0) = 0$  so  $C_{00}(t) = C_{00}(0)e^{-i\omega t/2} = 0$  for all time,  $C_{10}(0) = 1$ , and  $C_{0j}(0) = 0$  for all qubits  $j$ . As a function of time, we obtain

$$C_{10}(t) = e^{-i3\omega t/2} G_{10}(t) = \frac{1}{2} e^{-i3\omega t/2} (e^{\Gamma_1 t} + e^{\Gamma_2 t}) \quad (\text{Eq. 4})$$

With  $\Delta = 0$ , note that we have simply  $\Gamma_{1,2} = \pm i\sqrt{N}|\Omega_R|$ . Then

$$C_{10}(t) = \frac{1}{2} e^{-i3\omega t/2} (e^{i\sqrt{N}|\Omega_R|t} + e^{-i\sqrt{N}|\Omega_R|t}) = e^{-i3\omega t/2} \cos(\sqrt{N}|\Omega_R|t) \quad (\text{Eq. 5})$$

The probability a photon is excited as a function of time is  $P_{10}(t) = |C_{10}(t)|^2 = \cos^2(\sqrt{N}|\Omega_R|t)$ .

We similarly have as a function of time

$$C_{0j}(t) = i\Omega_R e^{-i3\omega t/2} \int_0^t \cos(\sqrt{N}|\Omega_R|\tau) d\tau = i \frac{\Omega_R}{\sqrt{N}|\Omega_R|} e^{-i3\omega t/2} \sin(\sqrt{N}|\Omega_R|t). \quad (\text{Eq. 6})$$

It follows that the probability as a function of time that the  $j$ th qubit is excited is  $P_{0j}(t) = |C_{0j}(t)|^2 = \frac{1}{N} \sin^2(\sqrt{N}|\Omega_R|t)$ . This makes physical sense because we can think of the probability that the  $j$ th qubit is excited as the probability that *any* qubit is excited, divided evenly  $N$  ways, which is what we have. It is worth noting that the probabilities sum to unity

$$P_{10}(t) + \sum_{j=1}^N P_{0j}(t) = \cos^2(\sqrt{N}|\Omega_R|t) + \sin^2(\sqrt{N}|\Omega_R|t) = 1, \quad (\text{Eq. 7})$$

as they should.

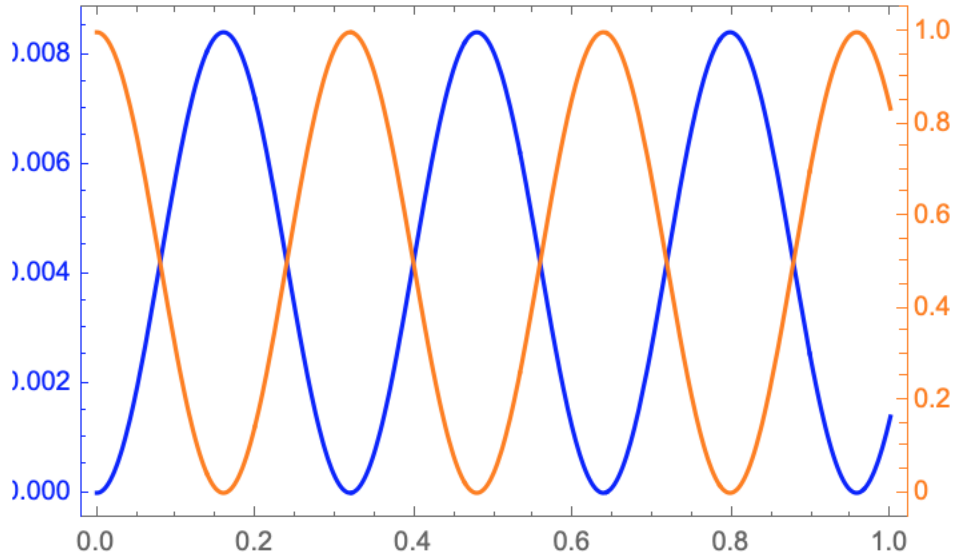


Figure 5.3: The orange plot shows  $P_{10}(t)$ , which has peaks at 1. The blue plot shows  $P_{0j}(t)$ , which only has peaks of just above 0.008 for any particular  $j$ . Time is in units of  $\text{meV}^{-1}$ .

Fig. 5.3 shows the probabilities as a function of time. Recall that this is a highly idealized

situation so no damping from outside noise has been considered which would cause the populations to all accumulate around the ground state after some amount of time. That is,  $P_{10}(t \rightarrow \infty) \rightarrow 0$  and  $P_{0j}(t \rightarrow \infty) \rightarrow 0$ .

### 5.3 Including noise

To see if strong coupling is a possibility, we must include relaxation rates of the system. The relaxation constants we denote  $\gamma_{n0}$  and  $\gamma_{n1}$  for the states with  $n$  photons and zero qubit excitations and  $n$  photons and 1 qubit excitation, respectively. If we take the system to be at zero temperature, these constants are given in [13] and are

$$\gamma_{n0} = \frac{\mu}{2}n, \quad \gamma_{n1} = \frac{\gamma}{2} + \frac{\mu}{2}n, \quad (\text{Eq. 1})$$

where  $\gamma$  and  $\mu$  are the partial relaxation rates atomic and photon field, respectively. These equations are a simplified version of the ones in [13] because we did not include phonons, quantized oscillations of the (in this case) graphene lattice, in this treatment.

To achieve strong coupling we must have that  $|\tilde{\Omega}_R| > \frac{\gamma_{10} + \gamma_{01}}{2}$ , where

$$\tilde{\Omega}_R = \sqrt{N|\Omega_R|^2 - (\gamma_{01} - \gamma_{10})^2/4}$$

by [13]. Squaring both sides of the inequality and rearranging gives the condition

$$N|\Omega_R|^2 > \frac{\gamma_{10}^2 + \gamma_{01}^2}{2} = \frac{\mu^2 + \gamma^2}{8}. \quad (\text{Eq. 2})$$

That is, with enough qubits depending on the partial relaxation rates  $\mu$  and  $\gamma$ , strong coupling can be achieved.

To put some numbers to this, suppose our decoherence is  $\gamma = 0.5$  meV. The left hand side of Eq. 2 is  $N|\Omega_R|^2 = 96.9$  meV<sup>2</sup>. In this case, the term  $\gamma^2/8$  works out to be .031 meV<sup>2</sup>. This allows for a relatively high  $\mu$ . Substituting in and neglecting the small  $\gamma$  term, we can see that the maximum  $\mu$  to allow for strong coupling is approximately 27 meV. Defining the quality of the

cavity to be  $Q = \omega/\mu$ , this gives a minimum allowable cavity quality of  $Q \approx 3.8$ , a very achievable number given that the quality of the cavity will increase as the reflectivity  $R$  increases, which we can do by increasing the number of pairs of alternating dielectrics  $M$ .

## 6. CONCLUSION

One way to create qubits is to apply a magnetic field perpendicular to a sheet of graphene. This creates Landau quantized graphene with a number of qubits that is proportional to the strength of the magnetic field. We can put this Landau quantized graphene in a plasmonic cavity and apply an optical electromagnetic field to it. These qubits will become entangled to the optical field. In some sense,  $N$  qubits couple to an optical field as if they are on qubit whose dipole moment and hence coupling strength is  $\sqrt{N}$  times larger.

The probability of the system being in a state with an excited optical mode and the probability of the system being in a state with one of the  $N$  qubits excited oscillate back and forth at a frequency  $\sqrt{N}|\Omega_R|$  with the system decaying into the ground state over a characteristic amount of time. If the coupling is stronger than this characteristic decay time, strong coupling can be achieved.

In the future, it would be interesting to obtain more exact numbers for the the partial relaxation rates to be able to calculate the system parameters, such as the reflectivity of the DBR, needed to ensure strong coupling in an experiment. Moreover, it would be interesting to include phonons, or lattice oscillations, coupling to the field-cavity system.



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