



Hybrid fuzzy and optimal modeling for water quality evaluation

Dong Wang,¹ Vijay P. Singh,² and Yuansheng Zhu³

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[1] Water quality evaluation entails both randomness and fuzziness. Two hybrid models are developed, based on the principle of maximum entropy (POME) and engineering fuzzy set theory (EFST). Generalized weighted distances are defined for considering both randomness and fuzziness. The models are applied to 12 lakes and reservoirs in China, and their eutrophic level is determined. The results show that the proposed models are effective tools for generating a set of realistic and flexible optimal solutions for complicated water quality evaluation issues. In addition, the proposed models are flexible and adaptable for diagnosing the eutrophic status.

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1. Introduction

[2] With growing human population and attendant human activities, eutrophication has recently become a severe problem in fresh as well as coastal waters in many regions of the world. Elevated inputs of nutrients can produce eutrophication that has been defined by the European Environment Agency (EEA) as “an increase in the rate of supply of organic matter to an ecosystem, which most commonly is related to nutrient enrichment enhancing the primary production in the system” [EEA, 2001]. The effective control of lake and reservoir eutrophication has therefore been attracting a great deal of interest these days [Somlyódy, 1998; Pei and Wang, 2003].

[3] Eutrophication of lakes and reservoirs is complicated by physical, chemical, and biological processes. It is known that water quality depends not only on natural processes, such as precipitation inputs, erosion and weathering of crustal material, and biota interrelationships, but also on anthropogenic influences, such as urban, industrial, and agricultural activities [Papatheodorou et al., 2006]. Interactions between the water body and its surrounding region are complex, but their analysis on the watershed scale is required for eutrophication assessment and management [Hession and Storm, 2000]. Furthermore, uncertainties, including randomness and fuzziness, are ubiquitous in water quality assessment and management. Issues of indeterminacy, heterogeneity, extremes, and fractal processes represent some of the most important and challenging environmental research topics [Anderson et al., 2000; Kirchner et al., 2000; Medina et al., 2002; Kirchner et al., 2004; Neal and Heathwaite, 2005].

[4] The importance of uncertainty in the water quality area is widely recognized and documented [e.g., Beck,

1987; Van der Perk, 1997; Beven and Freer, 2001; Beven, 2002; Vrugt et al., 2002; Harris and Heathwaite, 2005; Zheng and Keller, 2006]. Several common problems, such as the incompatibility of observations and the need for implicit value judgments, are hard to solve. Two basic and significant uncertainties during evaluation should be considered simultaneously in the development of water quality models. One is randomness, which is reflected in the monitoring and analysis of data related to eutrophication. The other is fuzziness, which is reflected in the evaluation of the classification standard, the evaluation class, and the degree of pollution. The trophic status classifications are well known to be fuzzy around their boundaries, and the relationships between the parameters [Vollenweider et al., 1998] are known to be uncertain. Thus there is a need to promote uncertainty estimation and to develop models and analytical tools [McIntyre et al., 2003]. In general, there are four approaches to investigate uncertainty as regards water quality evaluation in lakes and reservoirs: (1) the statistical and stochastic approach; (2) the fuzzy set approach; (3) the artificial intelligence (AI) approach; and (4) the hybrid approach. The above four approaches are data-based modeling approaches. Each approach must use high-quality data with explicit recognition of the spatial and temporal heterogeneity of hydrologic and environmental processes, in order to more consistently get right answers for right reasons [Kirchner, 2006]. Thus adequate and advanced measurements are vital to obtain such data.

[5] 1. In the statistical and stochastic approach, Shannon [1948] in a seminal contribution showed that information is statistical in nature. Many statistical methods, such as multivariate statistical techniques [Yu et al., 2003; Papatheodorou et al., 2006], are widely used to characterize water quality. Over the past 10 years, another statistical technique, the principal component analysis (PCA), has been widely used in determining eutrophication [Vega et al., 1998; Parinet et al., 2004].

[6] 2. Zadeh [1965] introduced fuzzy sets, which have been widely used in many fields and specify uncertainty by membership functions. Fuzzy set theory has been used to evaluate environmental quality. Silvert [2000] stated that

¹Department of Hydrosociences, Department of Earth Sciences, Nanjing University, Nanjing, China.

²Department of Biological and Agricultural Engineering, Texas A&M University, College Station, Texas, USA.

³College of Water Resources and Environment, Hohai University, Nanjing, China.

fuzzy logic could be applied to the development of environmental indices in a manner that solves several common problems. Many fuzzy methods have been used for water quality evaluation [Lu and Lo, 2002; Liou and Lo, 2005; Ghosh and Mujumdar, 2006]. Chen [1998] extended Zadeh's fuzzy set theory and named it the engineering fuzzy set theory (EFST) which provides a new way to ascertain the membership degree and membership function.

[7] 3. The artificial intelligence (AI) approach has recently been employed in a number of water quality applications, such as artificial neural network models (ANN) [Aguilera et al., 2001; Schulze et al., 2005] and genetic algorithm (GA) [Gentry et al., 2003; Kuo et al., 2006].

[8] 4. The hybrid approach combines two or more of the aforementioned approaches above to develop a hybrid model. It is a multiuse and powerful tool for modeling complex processes and characterizing uncertainty in water quality evaluation. Examples include combining the fuzzy sets theory and grey systems theory [Chang et al., 1996]; self-organizing maps and fuzzy sets theory [Lu and Lo, 2002]; ANN embedded Monte Carlo method [Zou et al., 2002]; and ANN and empirical models [Jain and Jha, 2005]. Another way to develop a hybrid model is to combine informational entropy theory, especially the principle of maximum entropy (POME), and engineering fuzzy set theory (EFST). As far as we know, no study has been reported using such a hybrid approach.

[9] To appreciate the usefulness of entropy, a short discussion is in order. Shannon [1948] developed a mathematical theory of entropy. Nearly a decade later, Jaynes [1957] formulated the principle of maximum entropy (POME). Entropy, especially POME and maximum entropy spectral analysis (MESA), has been applied in a wide range of areas, including hydrology and environmental and water resources [Singh, 1997]. For example, POME has been used to derive a variety of distributions and estimate their parameters [Singh, 1998]. Wang et al. [2004] used MESA for annual maximum series of tide level of the Changjiang River estuary. The state of the art of entropy applications in environmental and water resources has been reported by Singh [1997] and Wang and Zhu [2001].

[10] The government agencies must protect water bodies from getting polluted and treat polluted water. To that end they need to collect data and make decisions. In this kind of environmental decision-making, the following issues are normally of interest: (1) evaluation of the status of water quality in rivers, reservoirs, and estuaries; (2) pollution of water bodies and sources of pollution; (3) treatment of polluted waters; (4) reduction of pollution and pollution sources; (5) health effects of pollution; (6) ecosystem effects of pollution; (7) migration of pollutants; and (8) others. Comprehensive and reliable evaluation of the status of water quality is of fundamental importance, and only then can one clearly and accurately ascertain the status of the environment. It is therefore no surprise that this has been a growing area of research during the past years.

[11] The objective of this study is to improve the evaluation method and promote growing awareness of the need for properly incorporating uncertainty estimates into the understanding of environmental quality. To that end, a hybrid approach is proposed, based on the principle of maximum entropy (POME) and engineering fuzzy set

theory (EFST), considering both the randomness and the fuzziness in the evaluation and characterization of the water quality status. Two hybrid fuzzy and optimal models, named model I and model II, are developed and verified to determine the trophic state of 12 lakes and reservoirs in China as a case study with the use of official data. Both models exhibit that more information with less uncertainty during the water quality evaluation could be offered and used for better environmental decision-making. The theory used and the models developed here can also be applied to other areas.

2. Basic Concepts

2.1. Informational Entropy

[12] Shannon [1948] developed the entropy (informational) theory which numerically expresses a measure of uncertainty, $I[f]$ or $I[x]$, associated with the probability density function (pdf) $f(x)$ of any random variable X as

$$I[f] = - \int_a^b f(x) \ln[f(x)] dx. \quad (1)$$

Entropy allows choosing $f(x)$, which minimizes the uncertainty subject to specified constraints. Note that $f(x)$ is conditioned on the constraints used for its derivation.

2.2. Principle of Maximum Entropy (POME)

[13] For choosing the least biased probability distribution, $f(x)$, Jaynes [1957] formulated POME, which means that the minimally prejudiced assignment of probabilities is that which maximizes the entropy subject to the given information. Mathematically, it can be stated as follows: Given m linearly independent constraints C_i as

$$C_i = \int_a^b y_i f(x) dx, \quad i = 1, 2, \dots, m, \quad (2)$$

where $y_i(x)$ are some functions whose averages over $f(x)$ are specified, then the maximum of $I(f)$ subject to the conditions given by equation (2) is given as

$$f(x) = \exp \left[-\lambda_0 - \sum_{i=1}^m \lambda_i y_i(x) \right], \quad (3)$$

where λ_i , $i = 1, 2, \dots, m$, are the Lagrange multipliers, and can be determined from equations (2) and (3) along with the normalization condition:

$$\int_a^b f(x) dx = 1. \quad (4)$$

2.3. Fuzzy Sets Theory: Degree of Membership and Membership Function

[14] In the development of his fuzzy sets theory, Zadeh successfully introduced fuzzy sets (FS), fuzzy systems, fuzzy logic, linguistic variable and approximate reasoning, fuzzy information granulation, fuzzy logic and soft computing with words, fuzzy logic and perception-based theory, and the generalized theory of uncertainty (GTU). GTU

adopts a much more general conceptual structure in which statistical information is just one, albeit an important one, of many forms of information. The centerpiece of GTU is the concept of a generalized constraint, a concept drawn from fuzzy logic [Zadeh, 2005]. In the fuzzy set theory, a fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership varying between zero and one. A fuzzy set can be defined as follows:

[15] Let $X = \{x\}$ denote a space of points (objects), with x denoting a generic element of X . Then a fuzzy set A in X is a set of ordered pairs

$$X = \{[x, \mu_A(x)]\}, \quad x \in X, \quad (5)$$

where $\mu_A(x)$ is termed the grade of membership of x in A . Thus, if $\mu_A(x)$ takes on values in space M , termed the membership space, then A is essentially a function from X to M . The function $\mu_A : X \rightarrow M$, which defines A , is called the membership function of A . For simplicity, M is the interval $[0, 1]$, with grades 0 and 1 representing, respectively, nonmembership and full membership in a fuzzy set.

[16] For example, let $A = \{x|x \gg 1\}$ (that is, A is the fuzzy set of real numbers that are much larger than 1). Then, such a set may be defined subjectively by a membership function as equation (6):

$$\mu_A(x) = \begin{cases} 0 & \text{for } x \leq 1, \\ [1 + (x - 1)^{-2}]^{-1}, & \text{for } x > 1. \end{cases} \quad (6)$$

If $x \leq 1$, such as $x = 1$, then with the use of equation (6), $\mu_A(x = 1) = 0$, which means the grade of membership of $x(= 1)$ in $A(= \{x|x \gg 1\})$ is 0. If $x > 1$, such as $x = 10$, then with the use of equation (6), $\mu_A(x) = 0.987805$, which means the grade of membership of $x(= 10)$ in $A(= \{x|x \gg 1\})$ is 0.987805. While $x = 1000$, then with the use of equation (6), $\mu_A(x) = 0.999999$, which means the grade of membership of $x(= 1000)$ in $A(= \{x|x \gg 1\})$ is 0.999999.

2.4. Degree of Relative Membership and Relative Membership Function in Engineering Fuzzy Set Theory

[17] Chen [1998] extended Zadeh's fuzzy set theory and named it engineering fuzzy set theory (EFST), which provides a new way to ascertain the membership degree and membership function. Consider a fuzzy subset \tilde{A} in the domain of interest. Let two apices of \tilde{A} take on 0 and 1 to form a continuum on the closed interval $[0, 1]$. Then, for establishing a reference system on the number axis of this continuum, let a couple of arbitrary points of the reference system as the two apices of its coordinate take on 0 and 1. This leads to a reference continuum on the number axis $[0, 1]$ of the reference system. A number $\mu_{\tilde{A}}(u)$, $\forall u \in U$, is designated in the reference continuum and is named as the relative membership degree of u to \tilde{A} , and the following mapping is named as the relative membership function of \tilde{A} :

$$\left. \begin{aligned} &\mu_{\tilde{A}} : U \rightarrow [0, 1] \\ &u \mapsto \mu_{\tilde{A}}(u) \in [0, 1]. \end{aligned} \right\} \quad (7)$$

[18] For example, in the field of water quality evaluation, such as evaluation of eutrophication, two types of classification indices exist. One type of index is the descending classification index, such as chlorophyll a (chl a), total phosphorous (TP), total nitrogen (TN), and chemical oxygen demand (COD). The larger the values of these classification indices are, the higher the nutrient level is. The other type is the ascending classification index, such as Secchi disc depth (SD); the larger the values of these classification indices are, the lower the nutrient level is.

[19] As to the descending classification indices, if the relative membership degree is used, then the following is stated: (1) In class 1, the relative membership degree $s_{i1} = 0$, which is of the standard concentration y_{i1} of classification index i toward the evaluation concept A . (2) In class c , the relative membership degree $s_{ic} = 1$, which is of the standard concentration y_{ic} of classification index i toward this evaluation concept A . Then the relative membership degree denoted by s_{ih} , which is of the standard concentration y_{ih} of classification index i in class h , can be expressed with the use of a linear relation as

$$s_{ih} = \frac{y_{ih} - y_{i1}}{y_{ic} - y_{i1}}. \quad (8)$$

[20] It is the same for the ascending classification indices.

3. Hybrid Fuzzy and Optimal Evaluation Models for Eutrophication

3.1. Data Processing

[21] Let the number of classification standard for the evaluation of lake eutrophication be denoted by c , the number of evaluation classification index be denoted by m , and the concentration value of each index of the classification standard be denoted by y_{ih} . Then a matrix of concentration values can be constructed and denoted as $Y: [y_{ih}]_{m \times c}$ with dimensions of evaluation classification index m and classification standard c :

$$Y = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1c} \\ y_{21} & y_{22} & \cdots & y_{2c} \\ \dots & \dots & \dots & \dots \\ y_{m1} & y_{m2} & \cdots & y_{mc} \end{bmatrix} = [y_{ih}]_{m \times c}. \quad (9)$$

[22] For example, in China, Shu [1990] defined the trophic state as a function of nutrient levels, where chlorophyll a (chl a), total phosphorous (TP), total nitrogen (TN), chemical oxygen demand (COD) and Secchi disc depth (SD) are the classification indices, as shown in Table 1. Now there are n water samples for evaluation. In each sample there are monitored values x_{ij} of the m classification index. Then the matrix of monitored concentration values of these samples can be obtained as

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} = [x_{ij}]_{m \times n}. \quad (10)$$

Table 1. Trophic State as a Function of Nutrient Levels as Defined by *Shu* [1990] in China

Trophic State	Chl <i>a</i> , mg/m ³	TP, mg/m ³	TN, mg/m ³	COD, mg/L	SD, m
Oligotrophic	<1.0	<2.5	<30	<0.3	>10.0
Oligo-meso	2.0	5.0	50	0.4	5.0
Mesotrophic	4.0	25	300	2.0	1.5
Meso-eutro	10	50	500	4.0	1.0
Eutrophic	65	200	2000	10	0.4
Worse than eutrophic	>160	>600	>6000	>25	<0.3

[23] Using equation (8), the matrix of concentration values *Y* given by equation (9) for the evaluation of eutrophication and classification standard can be transformed to the matrix of concentration values of the relative membership degree with the classification standard $\tilde{S}: [s_{ih}]_{m \times c}$ as

$$\tilde{S} = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1c} \\ s_{21} & s_{22} & \cdots & s_{2c} \\ \dots & \dots & \dots & \dots \\ s_{m1} & s_{m2} & \cdots & s_{mc} \end{bmatrix} = [s_{ih}]_{m \times c}. \quad (11)$$

[24] A monitored value x_{ij} can be transformed to the corresponding relative membership degree r_{ij} as

$$r_{ij} = \begin{cases} 1 & x_{ij} > y_{ic} \\ \frac{x_{ij} - y_{i1}}{y_{ic} - y_{i1}} & y_{i1} \leq x_{ij} \leq y_{ic} \\ 0 & x_{ij} < y_{i1} \end{cases} \quad (12)$$

or

$$r_{ij} = \begin{cases} 1 & x_{ij} < y_{ic} \\ \frac{x_{ij} - y_{i1}}{y_{ic} - y_{i1}} & y_{i1} \geq x_{ij} \geq y_{ic} \\ 0 & x_{ij} > y_{i1} \end{cases} \quad (13)$$

[25] Consequently, the matrix of monitored values in the samples for evaluation, $X: [x_{ij}]_{m \times n}$, is transformed to the matrix of monitored values of the relative membership degree $\tilde{R}: [r_{ij}]_{m \times n}$ as

$$\tilde{R} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \dots & \dots & \dots & \dots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} = [r_{ij}]_{m \times n}. \quad (14)$$

[26] Furthermore, in order to determine the trophic state in China, five classification indices are used, namely, chlorophyll *a* (chl *a*), total phosphorous (TP), total nitrogen (TN), chemical oxygen demand (COD), and Secchi disc depth (SD), as shown in Table 1. Nevertheless each classification index takes on a different role in determining the nutrient level. Thus the influence weight of each evaluation classification index is introduced. For example, chl *a* index plays the most important role among the five, so the influence weight of the chl *a* index should be the maximum (max). The influence weight of each evaluation classification index can be denoted with the use of classification index weight vector \vec{v} as

$$\begin{cases} \vec{v} = (v_1, v_2, \dots, v_m) \\ \sum_{i=1}^m v_i = 1. \end{cases} \quad (15)$$

[27] Now the synthesis weight matrix $A: [v_i v_{ij}]_{m \times n}$ can be constructed as

$$A = \begin{bmatrix} v_1 & 0 & \cdots & 0 \\ 0 & v_2 & \cdots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \cdots & v_m \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \dots & \dots & \dots & \dots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} = \begin{bmatrix} v_1 r_{11} & v_1 r_{12} & \cdots & v_1 r_{1n} \\ v_2 r_{21} & v_2 r_{22} & \cdots & v_2 r_{2n} \\ \dots & \dots & \dots & \dots \\ v_m r_{m1} & v_m r_{m2} & \cdots & v_m r_{mn} \end{bmatrix} = [v_i r_{ij}]_{m \times n} \quad (16)$$

[28] Similarly, when *A* returns to 1 according to its column, the classification index synthesis weight matrix $W: [w_{ij}]_{m \times n}$, can also be constructed as

$$W = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \dots & \dots & \dots & \dots \\ w_{m1} & w_{m2} & \cdots & w_{mn} \end{bmatrix} = [w_{ij}]_{m \times n} \quad (17)$$

$$\begin{cases} w_{ij} = \frac{v_i r_{ij}}{\sum_{i=1}^m v_i r_{ij}} \\ \sum_{i=1}^m w_{ij} = 1 \quad j = 1, 2, \dots, n \end{cases} \quad (18)$$

[29] If the matrix of relative membership degrees of *n* samples toward class *c* is $\tilde{U}: [u_{hj}]_{c \times n}$,

$$\tilde{U} = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ u_{21} & u_{22} & \cdots & u_{2n} \\ \dots & \dots & \dots & \dots \\ u_{c1} & u_{c2} & \cdots & u_{cn} \end{bmatrix} = [u_{hj}]_{c \times n} \quad (19)$$

then the first constraint condition is

$$\begin{cases} \sum_{h=1}^c u_{hj} = 1 \\ u_{hj} \geq 0, \quad j = 1, 2, \dots, n \end{cases} \quad (20)$$

[30] Many matrices, such as $\tilde{U}: [u_{hj}]_{c \times n}$ consistent with equation (20), can be obtained. The objective of this study is to obtain an exclusive fuzzy optimal matrix $\tilde{U}: [u_{hj}]_{c \times n}$.

3.2. Hybrid Fuzzy and Optimal Evaluation Model I

[31] Matrix $\mathbf{U} : [u_{hj}]_{c \times n}$ entails both randomness and fuzziness. In fact, the randomness here occurs at least for two reasons: (1) randomness during evaluation, and (2) random error in observed values. Here the randomness during evaluation is considered using the concept of relative membership degree.

[32] The u_{hj} term denotes the relative membership degree of sample j belonging to class h . If u_{hj} is treated as the probability of sample j belonging to class h , then the Shannon entropy can be employed to assess this stochastic uncertainty as

$$H_j = - \sum_{h=1}^c u_{hj} \ln u_{hj}. \quad (21)$$

[33] The difference of sample j belonging to class h can be expressed as a generalized weighted distance ${}^1D_{hj}$ as

$${}^1D_{hj} = u_{hj} {}^1d_{hj} = u_{hj} \left\{ \sum_{i=1}^m [w_{ij}(r_{ij} - s_{ih})]^p \right\}^{\frac{1}{p}}, \quad (22)$$

where p is a distance parameter. When $p = 2$, it is named the Euclidean distance, which is often used.

[34] The $[u_{hj}]_{c \times n}$ matrix should minimize the sum of generalized weighted distances of whole samples to each class of the classification standard, namely,

$$\begin{aligned} \min_{u_{hj}} \quad & D = \sum_{j=1}^n \sum_{h=1}^c u_{hj} \left\{ \sum_{i=1}^m [w_{ij}(r_{ij} - s_{ih})]^p \right\}^{\frac{1}{p}} \\ \text{s.t.} \quad & \begin{cases} \sum_{h=1}^c u_{hj} = 1 \\ u_{hj} \geq 0, \quad j = 1, 2, \dots, n \end{cases} \end{aligned} \quad (23)$$

[35] On the basis of the principle of maximum entropy (POME), the $[u_{hj}]_{c \times n}$ matrix should also maximize the Shannon entropy, namely,

$$\begin{aligned} \max_{u_{hj}} \quad & H = \sum_{j=1}^n \left(- \sum_{h=1}^c u_{hj} \ln u_{hj} \right) \\ \text{s.t.} \quad & \begin{cases} \sum_{h=1}^c u_{hj} = 1 \\ u_{hj} \geq 0, \quad j = 1, 2, \dots, n \end{cases} \end{aligned} \quad (24)$$

[36] Equations (23) and (24) constitute a dual-objective programming problem, which can be solved using a weighting method, a constraint method, or a hybrid method. In this study, the weighting method was used. Thus a single-objective programming is constructed as follows:

$$\begin{aligned} \min_{u_{hj}} \quad & Y = {}^1D + \frac{1}{\eta_1} H = \sum_{j=1}^n \sum_{h=1}^c \\ & \cdot \left\{ u_{hj} \left\{ \sum_{i=1}^m [w_{ij}(r_{ij} - s_{ih})]^p \right\}^{\frac{1}{p}} + \frac{1}{\eta_1} u_{hj} \ln u_{hj} \right\} \\ \text{s.t.} \quad & \begin{cases} \sum_{h=1}^c u_{hj} = 1 \\ u_{hj} \geq 0, \quad j = 1, 2, \dots, n \end{cases} \end{aligned}, \quad (25)$$

where η_1 is the weighting factor.

[37] The optimal solution of equation (25) should be consistent with the Kuhn-Tucker conditions [Cohon, 1978]. By changing the value of η_1 , and solving equation (25) iteratively, a noninferior solution can be derived. The Lagrange function of this programming problem is

$$\begin{aligned} L(u_{hj}, \lambda_1) = & \sum_{j=1}^n \sum_{h=1}^c \left\{ u_{hj} \left\{ \sum_{i=1}^m [w_{ij}(r_{ij} - s_{ih})]^p \right\}^{\frac{1}{p}} + \frac{1}{\eta_1} u_{hj} \ln u_{hj} \right\} \\ & + \lambda_1 \left(\sum_{h=1}^c u_{hj} - 1 \right), \end{aligned} \quad (26)$$

where λ_1 is the Lagrange multiplier.

[38] By differentiating equation (26) with respect to u_{hj} and equating to zero, one obtains

$$\frac{\partial L}{\partial u_{hj}} = \left\{ \sum_{i=1}^m [w_{ij}(r_{ij} - s_{ih})]^p \right\}^{\frac{1}{p}} + \frac{1}{\eta_1} (\ln u_{hj} + 1) + \lambda_1 = 0. \quad (27)$$

Equation (27) results in

$$u_{hj} = \exp \left[-\eta_1 \left\{ \sum_{i=1}^m [w_{ij}(r_{ij} - s_{ih})]^p \right\}^{\frac{1}{p}} - \eta_1 \lambda_1 - 1 \right]. \quad (28)$$

[39] By differentiating equation (26) with respect to λ_1 and equating to zero, one obtains

$$\frac{\partial L}{\partial \lambda_1} = \sum_{h=1}^c u_{hj} - 1 = 0 \quad (29)$$

[40] Equation (28) and equation (29) yield

$$\exp[-(\eta_1 \lambda_1 + 1)] = \left(\sum_{h=1}^c \exp \left[-\eta_1 \left\{ \sum_{i=1}^m [w_{ij}(r_{ij} - s_{ih})]^p \right\}^{\frac{1}{p}} \right] \right)^{-1}. \quad (30)$$

[41] By inserting equation (30) in equation (28), one obtains

$$\begin{aligned} u_{hj} = & \exp \left[-\eta_1 \left\{ \sum_{i=1}^m [w_{ij}(r_{ij} - s_{ih})]^p \right\}^{\frac{1}{p}} \right] \\ & \cdot \left(\sum_{h=1}^c \exp \left[-\eta_1 \left\{ \sum_{i=1}^m [w_{ij}(r_{ij} - s_{ih})]^p \right\}^{\frac{1}{p}} \right] \right)^{-1}. \end{aligned} \quad (31)$$

[42] This is the hybrid fuzzy and optimal evaluation model based on the fuzzy set theory and entropy.

3.3. Hybrid Fuzzy and Optimal Evaluation Model II

[43] The difference of sample j belonging to class h can be also expressed as generalized weighted distances ${}^2D_{hj}$ as

$${}^2D_{hj} = u_{hj} {}^2d_{hj} = u_{hj} \left[\sum_{i=1}^m (w_{ij} |r_{ij} - s_{ih}|) \right]. \quad (32)$$

In the same way as above, a single-objective programming is constructed as

$$\begin{aligned} \min_{u_{hj}} \quad & Y = {}^2D + \frac{1}{\eta_2} H \\ & = \sum_{j=1}^n \sum_{k=1}^c \left\{ u_{hj} \left[\sum_{i=1}^m (w_{ij} |r_{ij} - s_{ih}|) \right] + \frac{1}{\eta_2} u_{hj} \ln u_{hj} \right\}, \\ \text{s.t.} \quad & \begin{cases} \sum_{h=1}^c u_{hj} = 1 \\ u_{hj} \geq 0, \quad j = 1, 2, \dots, n \end{cases} \end{aligned} \quad (33)$$

where η_2 is the weighting factor.

[44] The Lagrange function of this programming problem is

$$\begin{aligned} L(u_{hj}, \lambda_2) = & \sum_{j=1}^n \sum_{h=1}^c \left\{ u_{hj} \left[\sum_{i=1}^m (w_{ij} |r_{ij} - s_{ih}|) \right] + \frac{1}{\eta_2} u_{hj} \ln u_{hj} \right\} \\ & + \lambda_2 \left(\sum_{h=1}^c u_{hj} - 1 \right), \end{aligned} \quad (34)$$

where λ_2 is the Lagrange multiplier.

[45] Differentiating equation (34) with respect to u_{hj} and equating to zero, one obtains

$$\frac{\partial L}{\partial u_{hj}} = \sum_{i=1}^m (w_{ij} |r_{ij} - s_{ih}|) + \frac{1}{\eta_2} (\ln u_{hj} + 1) + \lambda_2 = 0. \quad (35)$$

Equation (35) results in

$$u_{hj} = \exp \left[-\eta_2 \sum_{i=1}^m (w_{ij} |r_{ij} - s_{ih}|) - \eta_2 \lambda_2 - 1 \right]. \quad (36)$$

[46] Differentiating equation (34) with respect to λ_2 and equating to zero, one obtains

$$\frac{\partial L}{\partial \lambda_2} = \sum_{h=1}^c u_{hj} - 1 = 0. \quad (37)$$

[47] Equation (36) and equation (37) yield

$$\exp[-(\eta_2 \lambda_2 + 1)] = \left(\sum_{h=1}^c \exp \left[-\eta_2 \sum_{i=1}^m (w_{ij} |r_{ij} - s_{ih}|) \right] \right)^{-1}. \quad (38)$$

[48] Inserting equation (38) in equation (36), one obtains

$$\begin{aligned} u_{hj} = & \exp \left[-\eta_2 \sum_{i=1}^m (w_{ij} |r_{ij} - s_{ih}|) \right] \\ & \cdot \left(\sum_{h=1}^c \exp \left[-\eta_2 \sum_{i=1}^m (w_{ij} |r_{ij} - s_{ih}|) \right] \right)^{-1}. \end{aligned} \quad (39)$$

[49] This is the hybrid fuzzy and optimal evaluation model II.

4. Application

4.1. Study Sites

[50] For the past three decades, enormous changes have been occurring at virtually all levels in China. The increase in population and the need for economic growth have inevitably accelerated more development projects, especially

water resources development and utilization projects. However, the rapid pace of development has environmental consequences. For example, pollution in waterways and reservoirs has increased significantly. For example, for 12 lakes and reservoirs employed in this study, monitored values of eutrophication indices are listed in Table 2 [Shu, 1990]. These lakes are important to the local environment and human lives in China.

4.2. Data

[51] In China the Ministry of Water Resources of China and State Environmental Protection Administration of China have professionals whose primary responsibility is to collect water quality data. These data are subject to a variety of national standards and professional standards, such as those of the *State Environmental Protection Administration of China* [2002] and of *Ministry of Water Resources of China* [1999, 1998, 1994], etc., which assure and control the quality of the data. These standards also regulate sampling, monitoring, inspecting, analyzing, and experimenting of water quality data in a comprehensive manner. The data used in this study are mainly from branches of the Ministry of Water Resources of China and the State Environmental Protection Administration of China. These data have been used for a variety of research endeavors, such as the discussion of methods for evaluation of eutrophication of Chinese lakes [Shu, 1990], the use of fuzzy set theory for the assessment for lake and reservoir eutrophication [Chen and Xiong, 1993], and so on.

4.3. Application of Models

[52] Using the monitored values of eutrophication indices of lakes and reservoirs in Table 2, step by step application of the models is outlined: (1) With the use of equation (8), s_{ih} is obtained. (2) With the use of equations (12) and (13), r_{ij} is obtained. (3) With the use of equations (16), (17), and the following v , w_{ij} is obtained:

$$\vec{v} = \left[\frac{|r_1|}{\sum_{i=1}^m r_i}, \frac{|r_2|}{\sum_{i=1}^m r_i}, \dots, \frac{|r_m|}{\sum_{i=1}^m r_i} \right],$$

where r_1, r_2, \dots, r_m is the correlation coefficient of the index m to chlorophyll a (chl a). The correlation coefficient of the index chl a to itself is 1, and then its weight is greater than any other from the above equation. Here the same classification index weight vector \vec{v} as Shu's [1990] is adopted as $\vec{v} = (0.233, 0.217, 0.189, 0.210, 0.151)$. (4) With the use of equation (31), taking $p = 2$, which means the Euclidean distance, the values of the hybrid fuzzy and optimal evaluation model I are obtained and showed in Table 3.

[53] Taking sample 6 as an example, the relative membership degree of 6 to the trophic state O (oligotrophic) is 0, the relative membership degree to the trophic state O-M (oligo-meso) is 0.0003, the relative membership degree to the trophic state M (mesotrophic) is 0.0710, the relative membership degree to the trophic state M-E (meso-eutro) is 0.1610, the relative membership degree to the trophic state

Table 2. The Monitored Values of Eutrophication Indices of Lakes and Reservoirs in China^a

Number	Name of Lake or Reservoir	Chl <i>a</i> , mg/m ³	TP, mg/m ³	TN, mg/m ³	COD, mg/L	SD, m
1	Qionghai	0.88	130	410	1.43	2.98
2	Erhai	4.33	21	180	3.38	2.40
3	Bositeng Lake	4.91	50	969	5.42	1.46
4	Yuqiao Reservoir	16.20	26	1,020	5.16	1.16
5	Cihu Lake	15.38	87	1,540	4.40	0.65
6	Chaohu Lake	14.56	140	2,270	4.34	0.27
7	Gantang Lake	77.70	135	2,140	6.96	0.36
8	Mogu Lake	82.40	332	2,660	14.60	0.49
9	West Lake in Hangzhou	95.94	136	2,230	10.18	0.37
10	Xuanwu Lake in Nanjing	202.10	708	6,790	8.86	0.31
11	Moshui Lake in Wuhan	262.40	500	16,050	13.60	0.15
12	Dongshan Lake in Guangzhou	185.10	670	7,200	14.80	0.26

^aFrom *Shu* [1990].

E (eutrophic) is 0.7668, and the relative membership degree to the trophic state W-E (worse than eutrophic) is 0.0009. Thus the trophic state of 6 is eutrophic using the fuzzy and optimal evaluation model I.

[54] In step 4 using equation (39), the values of the hybrid fuzzy and optimal evaluation model II are also obtained and shown in Table 4.

4.4. Model Verification

[55] The results of evaluation of eutrophication obtained by the use of hybrid fuzzy and optimal models I and II are given in Table 5. The results of evaluation with the use of the fuzzy model [*Chen and Xiong*, 1993], in which only fuzziness is taken into account, are also given in Table 5. The results produced from model I and model II are almost the same, and the results produced from these two models are consistent with field surveys.

[56] Contrasting with *Chen and Xiong's* fuzzy model, the results produced from model I and model II are more informative. For example, for samples 2, 4, and 6, models I and II provide more information about the eutrophication status. With these two models, the trophic state of 2 is M (mesotrophic) and partial M-E (meso-eutro), while with *Chen and Xiong's* fuzzy model, the trophic state is just M (mesotrophic). The trophic state of 4 is M-E (meso-eutro)

and partial M (mesotrophic) using these two models, while with *Chen and Xiong's* fuzzy model, the trophic state is just M-E (meso-eutro). The trophic state of 6 is E (eutrophic) and partial M-E (meso-eutro) using these two models, while with *Chen and Xiong's* fuzzy model, the trophic state is just E (eutrophic).

[57] The Shannon entropy of each model is given in Table 6. The Shannon entropy values of model I and model II are less than *Chen and Xiong's* fuzzy model, except for samples 4, 5, and 6. This means less uncertainty and more reliability of models I and II.

4.5. Discussion of Results

[58] Eutrophication has recently become a severe problem in the world. The government agencies need to make decisions to protect and treat water bodies. That is the reason that an accurate, comprehensive, and reliable evaluation of the status of water quality takes on an added importance and this is a growing area of research these days. Uncertainties, including randomness and fuzziness, are ubiquitous in water quality assessment and management. This study improves the evaluation method and advances the awareness of the need for properly incorporating uncertainty estimates into the understanding of environmental

Table 3. The Values of the Hybrid Fuzzy and Optimal Evaluation Model I for 12 Lakes and Reservoirs in China^a

Number	Name of Lake or Reservoir	O	O-M	M	M-E	E	W-E
1	Qionghai	0	0.1470	0.5918	0.2118	0.0487	0.0007
2	Erhai	0	0.0050	0.7875	0.1815	0.0239	0.0021
3	Bositeng Lake	0	0.0003	0.4777	0.4499	0.0717	0.0004
4	Yuqiao Reservoir	0	0.0001	0.3403	0.5928	0.0664	0.0003
5	Cihu Lake	0	0.0002	0.1866	0.4860	0.3264	0.0007
6	Chaohu Lake	0	0.0003	0.0710	0.1610	0.7668	0.0009
7	Gantang Lake	0	0.0006	0.0132	0.0258	0.9595	0.0008
8	Mogu Lake	0	0.0014	0.0078	0.0176	0.9613	0.0118
9	West Lake in Hangzhou	0	0.0007	0.0065	0.0133	0.9775	0.0019
10	Xuanwu Lake in Nanjing	0	0	0	0	0.0002	0.9998
11	Moshui Lake in Wuhan	0	0	0	0	0.0012	0.9988
12	Dongshan Lake in Guangzhou	0	0	0	0	0.0003	0.9997

^aAbbreviations of trophic states: O, oligotrophic; O-M, oligo-meso; M, mesotrophic; M-E, meso-eutro; E, eutrophic; W-E, worse than eutrophic.

Table 4. Values of the Hybrid Fuzzy and Optimal Evaluation Model II for 12 Lakes and Reservoirs in China^a

Number	Name of Lake or Reservoir	O	O-M	M	M-E	E	W-E
1	Qionghai	0	0.1671	0.4902	0.2770	0.0655	0.0002
2	Erhai	0	0.0274	0.6622	0.2875	0.0225	0.0004
3	Bositeng Lake	0	0.0028	0.4936	0.4552	0.0484	0
4	Yuqiao Reservoir	0	0.0020	0.3234	0.6315	0.0430	0.0001
5	Cihu Lake	0	0.0017	0.1849	0.5747	0.2385	0.0002
6	Chaohu Lake	0	0.0011	0.0752	0.2250	0.6982	0.0005
7	Gantang Lake	0	0.0004	0.0105	0.0321	0.9576	0.0003
8	Mogu Lake	0	0.0002	0.0024	0.0076	0.9849	0.0049
9	West Lake in Hangzhou	0	0.0002	0.0038	0.0116	0.9893	0.0005
10	Xuanwu Lake in Nanjing	0	0	0	0	0.0001	0.9999
11	Moshui Lake in Wuhan	0	0	0	0	0.0005	0.9995
12	Dongshan Lake in Guangzhou	0	0	0	0	0.0001	0.9999

^aAbbreviations of trophic state: O, oligotrophic; O-M, oligo-meso; M, mesotrophic; M-E, meso-eutro; E, eutrophic; W-E, worse than eutrophic.

quality. Thus more information with less uncertainty can be offered and used for better environmental decision making.

[59] More attention should be paid to two issues:

[60] 1. The first issue is data. The hybrid approach as well as the other three approaches mentioned in the introduction section are all data-based modeling approaches. They must be based on high-quality data with explicit recognition of spatial and temporal heterogeneity of hydrologic and environmental processes. To that end, adequate and accurate measurements are vital. On the other hand, assessment of data uncertainty and its effect on the evaluation should be emphasized more and more during each assessment and management. Otherwise, regardless of advances being made in modeling approaches, it may still be possible to get the “right answer for the wrong reasons” [Kirchner, 2006]. For example, in the case of the stochastic observation error of the data, most studies merely take the influence of physical weight of the observed index on the water-environment evaluation into account, but the influence of stochastic observation error is ill-considered. It has been found that the stochastic observation error of the data can even change the grades of evaluation in another study.

[61] 2. The second issue is subjectivity. Randomness and fuzziness are two uncertainties in water quality assessment and management. The trophic status classifications are well

known to be fuzzy around their boundaries, and the relationships between the parameters [Vollenweider *et al.*, 1998] are known to be stochastic. Since the assessment results obtained from considering only one uncertainty may easily mislead or bias the user, this study provides a useful approach to integrating randomness and fuzziness to determine the trophic status of lakes and reservoirs. At the same time, subjectivity should be given special attention. For example, one of the main reasons that people are concerned about lake water quality is offensive odors, while bad smells from a lake may or may not be due to water quality problems per se. Thus the value of an objective measure of eutrophic status has some inherently subjective components. Such subjective judgments involved represent an important and a challenging environmental research topic.

5. Conclusions

[62] Considering both randomness and fuzziness in water quality evaluation, generalized weighted distances are defined, and models for quality evaluation based on the principle of maximum entropy (POME) and engineering fuzzy set theory (EFST) are developed. Application of the models to the determination of the trophic level of 12 lakes and reservoirs in China shows that the proposed models are

Table 5. Comparison of Eutrophication Evaluation of Lakes and Reservoirs in China Using Fuzzy and Optimal Model I and II With Fuzzy Model of *Chen and Xiong* [1993]^a

Number	Name of Lake or Reservoir	Hybrid Fuzzy and Optimal Model I	Hybrid Fuzzy and Optimal Model II	Fuzzy Model of <i>Chen and Xiong</i> [1993]
1	Qionghai	M	M	M
2	Erhai	M (partial M-E)	M (partial M-E)	M
3	Bositeng Lake	M (partial M-E)	M (partial M-E)	M-E (partial M)
4	Yuqiao Reservoir	M-E (partial M)	M-E (partial M)	M-E
5	Cihu Lake	M-E (partial E)	M-E (partial E)	M-E (partial E)
6	Chaohu Lake	E (partial M-E)	E (partial M-E)	E
7	Gantang Lake	E	E	E
8	Mogu Lake	E	E	E
9	West Lake in Hangzhou	E	E	E
10	Xuanwu Lake in Nanjing	W-E	W-E	W-E
11	Moshui Lake in Wuhan	W-E	W-E	W-E
12	Dongshan Lake in Guangzhou	W-E	W-E	W-E

^aAbbreviations of trophic state: O, oligotrophic; O-M, oligo-meso; M, mesotrophic; M-E, meso-eutro; E, eutrophic; W-E, worse than eutrophic.

Table 6. Comparison of Shannon Entropy of Each Evaluation Model

Number	Name of Lake or Reservoir	Shannon Entropy of Hybrid Fuzzy and Optimal Model I	Shannon Entropy of Hybrid Fuzzy and Optimal Model II	Shannon Entropy of Fuzzy Model of Chen and Xiong [1993]
1	Qionghai	1.0730	1.1852	1.3985
2	Erhai	0.6261	0.8186	0.8292
3	Bositeng Lake	0.9071	0.8706	0.9128
4	Yuqiao Reservoir	0.8607	0.8040	0.6508
5	Cihu Lake	1.0361	0.9851	0.6631
6	Chaohu Lake	0.6942	0.7921	0.5798
7	Gantang Lake	0.2017	0.2030	0.2964
8	Mogu Lake	0.2091	0.0943	0.5683
9	West Lake in Hangzhou	0.1300	0.0944	0.4186
10	Xuanwu Lake in Nanjing	0.0018	0.0008	0.2542
11	Moshui Lake in Wuhan	0.0094	0.0043	0.4804
12	Dongshan Lake in Guangzhou	0.0030	0.0008	0.1742

an effective tool for diagnosing the eutrophic status of lake waters. From the results of this study, the following conclusions are drawn:

[63] 1. Although the evaluation of eutrophication of very different waters is hard to achieve, the two fuzzy and optimal models generate more acceptable alternatives, which are useful for objectively determining the trophic level.

[64] 2. The hybrid approach is appropriate for taking both randomness and fuzziness into account in the water quality evaluation and to characterize a highly uncertain, heterogeneous, and dynamic water-environmental system. As the case study shows, the hybrid approach can provide more information than considering just one uncertainty, such as fuzziness.

[65] 3. The hybrid approach possesses smaller uncertainty and more reliability as indicated by the Shannon entropy of each model.

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V. P. Singh, Department of Biological and Agricultural Engineering, Texas A&M University, 321 Scoates Hall, 2117 TAMU, College Station, TX 77843-2117, USA. (vsingh@tamu.edu)

D. Wang, Department of Hydrosociences, Department of Earth Sciences, Nanjing University, Nanjing, 210093, China. (wangdong@nju.edu.cn)

Y. Zhu, College of Water Resources and Environment, Hohai University, Nanjing, 210098, China. (zhuyuanheng@hotmail.com)