OPTIMIZATION OF A PETROLEUM PRODUCING ASSETS PORTFOLIO:

DEVELOPMENT OF AN ADVANCED COMPUTER MODEL

A Thesis

by

GIZATULLA AIBASSOV

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2007

Major Subject: Petroleum Engineering

OPTIMIZATION OF A PETROLEUM PRODUCING ASSETS PORTFOLIO:

DEVELOPMENT OF AN ADVANCED COMPUTER MODEL

A Thesis

by

GIZATULLA AIBASSOV

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Approved by:

Chair of Committee,	W. John Lee
Committee Members,	Duane A. McVay
	Maria A. Barrufet
Head of Department,	Stephen A. Holditch

December 2007

Major Subject: Petroleum Engineering

ABSTRACT

Optimization of a Petroleum Producing Assets Portfolio:

Development of an Advanced Computer Model. (December 2007)

Gizatulla Aibassov, B.Sc., Kazakh National University, Kazakhstan

Chair of Advisory Committee: Dr. W. John Lee

Portfolios of contemporary integrated petroleum companies consist of a few dozen Exploration and Production (E&P) projects that are usually spread all over the world. Therefore, it is important not only to manage individual projects by themselves, but to also take into account different interactions between projects in order to manage whole portfolios. This study is the step-by-step representation of the method of optimizing portfolios of risky petroleum E&P projects, an illustrated method based on Markowitz's Portfolio Theory. This method uses the covariance matrix between projects' expected return in order to optimize their portfolio.

The developed computer model consists of four major modules. The first module generates petroleum price forecasts. In our implementation we used the price forecasting method based on Sequential Gaussian Simulation. The second module, Monte Carlo, simulates distribution of reserves and a set of expected production profiles. The third module calculates expected after tax net cash flows and estimates performance indicators for each realization, thus yielding distribution of return for each project. The fourth module estimates covariance between return distributions of individual projects and compiles them into

portfolios. Using results of the fourth module, analysts can make their portfolio selection decisions.

Thus, an advanced computer model for optimization of the portfolio of petroleum assets has been developed. The model is implemented in a MATLAB® computational environment and allows optimization of the portfolio using three different return measures (NPV, GRR, PI). The model has been successfully applied to the set of synthesized projects yielding reasonable solutions in all three return planes. Analysis of obtained solutions has shown that the given computer model is robust and flexible in terms of input data and output results. Its modular architecture allows further inclusion of complementary "blocks" that may solve optimization problems utilizing different measures (than considered) of risk and return as well as different input data formats.

DEDICATION

To my beloved parents who have always been at my side in any endeavor, supportive

and, at times, critical.

ACKNOWLEDGEMENTS

First of all, I would like to take this opportunity to express my sincere appreciation to the chair of my graduate advisory committee, Dr. W. John Lee, for his guidance and support during the conducting and completion of this thesis project. It has been a pleasure to work with him.

I thank Dr. Duane McVay and Dr. Maria Barrufet for agreeing to be on my advisory committee and reviewing this work. I also thank Nazym Madaliyeva for her support while completing this project.

Finally, I thank the Government of the Republic of Kazakhstan for sponsoring my graduate program here at Texas A&M. Aggieland has definitely taught me many important things. Thank you, Aggieland, for being so illuminative. I will miss you.

Gig 'em!

NOMENCLATURE

E&P	Exploration and Production
PV	Present Value
SGS	Sequential Gaussian Simulation
WTI	West Texas Intermediate
CPI	Consumer Price Index
$z(t_a)$	Variogram Price Value at Time t_a
$\boldsymbol{h}$	Variogram Lag Step Size
$N(\boldsymbol{h})$	All Observations $\boldsymbol{h}$ Time Units Apart from Each Other
$\gamma^*(h)$	Estimated Semivariogram for Lag $\boldsymbol{h}$
OK	Ordinary Kriging
$z_1 \ldots z_n$	Ordinary Kriging Attribute Values at Location $x_1 \ldots x_n$
$\hat{Z}(x^*)$	Expected Attribute Value at Unobserved Location x^*
σ_{OK}^2	Ordinary Kriging Interpolation Error
OOIP	Original Oil in Place
N	Reserves Volume
ϕ	Porosity
S_w	Formation Water Saturation
h	Formation Thickness
A	Drainage Area
B_0	Formation Volume Factor

R_E	Recovery Efficiency Coefficient
k	Permeability
ρ	Correlation Coefficient
n	Number of Iterations
a	Production Decline rate
a_i	Initial Decline Rate
b	Hyperbolic Exponent
q_{ot}	Production Rate at Time t_0
q_i	Initial Production Rate
q_{EL}	Economic Limit
N_p	Ultimate Production
EOR	Enhanced Oil Recovery
Min	Minimum
Max	Maximum
CAPEX	Capital Expenditures
OPEX	Operating Expenses
G&G	Geological and Geophysical
PI	Profitability Index
NPV	Net Present Value
NCF_t	After Tax Net Cash Flow at Time t
i_d	Discount Rate
GRR	Growth Rate of Return

FV_t	Future Value of All Positive Net Cash Flows at Time t
PV_0	Present Value of All Negative Net Cash Flows at Time Zero
r	Vector of Project Returns
Cov	Projects' Return Covariance Matrix
σ_i^2	Variance of Asset i's Return
σ_{ij}	Covariance between Assets i and j
x	Projects' Weights Vector
R_p	Portfolio Return
σ_p^2	Portfolio Return Variance
W_i	Weight of Project i in Portfolio
IN_i	Total Cost of Project i
f_i	Investors' Percentage of Participation in Project i
TW	Total Cost of the Portfolio

TABLE OF CONTENTS

Page

Page

LIST OF FIGURES

LIST OF TABLES

CHAPTER I

INTRODUCTION

Currently, worldwide petroleum business is the business of large, integrated, multi-level and multi-national players. Incorporation and mergers are the common trend in the industry. According to Slocum (Slocum, 2001), as a result of recent mergers, the five largest oil companies operating in the United States now control 41% of domestic oil exploration and production. When their international oil production is included, these five corporations control 15% of the world's oil production.

Contemporary integrated petroleum companies do not operate in one or few fields anymore. Now their portfolios consist of few dozen E&P projects and usually spread all over the world. Petroleum producing projects are now considered as quite liquid assets, with their own worldwide market where they are readily bought and sold by the market participants. Thus, now it is important not only to manage individual projects by themselves, but also taking into account different interactions between projects, to manage the whole portfolios.

From financial point of view petroleum producing assets and conventional financial assets (such as stocks) have a great deal of similarities. Their cash flow profiles consist of the same major elements. First, an investor pays the purchase price, the role of which in case of petroleum assets plays PV of exploration and development investment.

_____

This thesis follows the style of *SPE Production & Operations*.

Then comes the annual stream of dividends, which is similar to the stream of annual after tax cash inflows from sale of oil. After some period of time, investors sell their stocks, thus receiving selling price. At the similar stage mature petroleum fields may also be sold for some selling price to independents, or they are abandoned with abandonment cost. In both cases, the purchase price that investors are willing to pay for the asset is defined by the future cash inflows that they are expecting to receive. Thus the process of asset pricing in both cases is mainly a process of estimation of future cash flow streams and their consequent discounting down to the present.

However, there are also some differences. First of all, this process of cash flow estimation is based on different input data. In the case of financial assets, their historical return is the best approximation of their future performance, while petroleum assets' cash flow is estimated using production forecast and petroleum price forecast (Ball & Savage, 1999). Second, financial asset investors do not get their return back until they sell the stock. Thus, stocks are bought to be sold, with dividends playing only minor role in overall stock return. Petroleum assets, however, are "bought" for their annual "dividends" (annual after tax sales revenues), with consequent sale of mature field being less important. Another important difference is that petroleum assets are much less divisible than financial assets; moreover their market is less efficient.

Taking into account all these similarities and differences, it is no surprise that management of a portfolio that consists of petroleum producing assets is, in general, similar to that of financial assets, with some industry specific differences.

Contemporary portfolio management ideas mainly rely on the theory developed by Harry Markowitz back in 1950s. Markowitz's *Portfolio Theory* (Markowitz, 1952) at the very beginning was applied to manage portfolio of financial assets. The main goal was, by means of changing fraction of total wealth invested in different individual assets that compose portfolio, to come up with an efficient portfolio that has the highest level of expected return for the given level of uncertainty of that return, that is, risk. Since then, Markowitz's bright ideas have been intensively used by investment managers in financial markets. Much later, with the growth of oil companies, the increasing size of their portfolios has caused a need for more knowledgeable management in the petroleum industry.

Now, an integrated approach to petroleum projects management, i.e. simultaneous management of the whole portfolios, is becoming increasingly popular in the industry, especially after works of Edwards and Hewett (1993), Hightower and David (1991), and others. Nonetheless this subject is still relatively new for the industry and not much literature is available, in particular, on step by step illustration of development of portfolio optimization model in petroleum industry context. Moreover with recent advancements in oil price uncertainty quantification methods, such as price forecasting method based on Sequential Gaussian Simulation (SGS) algorithm (Holmes *et al.*, 2006), application of these advancements to solve portfolio management problems is believed to be the logical development of the subject. Therefore the objectives of this study were as follows:

1. Develop an advanced computer model for optimization of portfolio consisting of petroleum E&P projects. The model should be flexible and not have any constraints in terms of the number of projects in portfolio, types of risk and return measures employed and apply best known production and price forecasting techniques.

2. Implement the SGS price forecasting method and apply it in portfolio selection model.

3. Illustrate and explain every step of the method of building such a model, for further reference of industry practitioners to this study.

4. Analyze and knowledgeably explain obtained results.

CHAPTER II

LITERATURE REVIEW

2.1 Financial Portfolio Management

The subject of selecting the portfolio of producing petroleum assets for a long time has not been getting as much attention from the side of financial advisors as it deserves. Specific enormous industry risks have not been taken into consideration, and many authors (Downey, 1997) advised to form the portfolio by ranking available projects according to their expected profitability index, and then marching down the list until the investor would run out of the money.

In the case of absolute certainty, this approach would give to the investor *"the biggest bang for the buck"* but it leaves out of the analysis one very important variable – the risk of actual cash flows not being equal to expected ones. Due to a set of different reasons related to the field, company, industry, and overall worldwide economy, actual cash flows are never equal to expected ones, and this uncertainty should be incorporated into the decision of selecting company portfolio.

For the first time, the subject of selecting the portfolio out of a set of available investments under conditions of uncertain returns was thoroughly considered by Markowitz (1952). In his revolutionizing work, Markowitz introduced major axioms and concepts that formed a framework of modern financial portfolio theory. First of all, he quantitatively defined risk as the statistical variance of actual returns from their mean or expected value. Second, Markowitz showed that the abovementioned profitability index

rule must be rejected, because it does not prefer diversified portfolios as opposed to undiversified ones. Third, he showed that diversification is the result of negative and positive correlations between the projects. And most importantly, Markowitz introduced a new rule of portfolio selection: a risk adverse investor would require more return for the given level of assumed risk and would want to bear less risk for the given level of expected return. This rule redefined the portfolio selection problem into an optimization problem: maximize return for the given level of risk, or vice versa: minimize risk for the given level of return, by changing amounts of money invested in different investment opportunities available to the company.

According to Markowitz's ideas, every portfolio that has its unique expected return and variance of return (i.e. risk) combination can be represented as a point on the risk/return plain. The whole set of portfolios available to the investor (opportunity set) is represented as a plain of individual points. Then while selecting our portfolio we would consider only the upper-left border of opportunity set, called the "efficient set" because all the portfolios in this set are efficient; that is, they dominate over all other available portfolios in the opportunity set either by offering more return for the given level of risk or by assuming less risk for the given level of return.

Sharpe (1964) further enhanced Markowitz's ideas. He showed that for assumptions of *an* efficient market if we would include in our analysis an asset that does not bear any risk and thus provides risk-free return, then the efficient frontier would change its shape from curved into the straight line called the capital market line. Moreover, the knowledgeable investor would select only that portfolio of risky assets,

called the market portfolio, which is represented by the point of tangency between the capital market line and the opportunity set, because the market portfolio would provide the straight efficient frontier with the highest slope, that is, the highest return for a unit of assumed risk.

In this case of selecting the portfolio with the risk-free asset, the portfolio selection problem would change into selecting any point along the straight line by changing the amount of money invested in the market portfolio and the risk-free asset. Points that lie between the market portfolio and the risk-free asset can be achieved by investing part of the money into risk free asset and another part into market portfolio, while points that lie beyond market portfolio can be achieved by borrowing money at the risk-free rate, and investing all available money into market portfolio.

2.2 Petroleum E&P Portfolio Management

All the concepts presented earlier were developed for financial assets with the assumption of efficient market conditions. In 1968, David B. Hertz (Hertz, 1968) discussed the application of the Markowitz's model to risky industrial projects as opposed to stocks. Much later, Ball and Savage (1999) showed that major differences between financial assets and petroleum producing assets are:

1.   Risk structure of stock portfolios is different that that of petroleum assets. E&P projects face both local uncertainties involving discovery and production of oil at a given site, and global uncertainties involving price, policies, etc.

2.   Risk in stock portfolios is usually measured in terms of volatility, while E&P portfolios must specifically track downside risk.

3.   The stock market is quite efficient whereas the market of E&P projects is inefficient.

4.   A stock portfolio generally contains a small fraction of the outstanding shares of any one company, while E&P portfolio often contain 100% of its constituent projects, creating budgetary effects.

Moreover, taking into account all these differences, Ball and Savage presented their interpretation of Markowitz's portfolio model for application with petroleum E&P assets and called it the E&P portfolio optimization model.

There were a number of other attempts of readjusting and applying Markowitz's portfolio model for petroleum industry needs. Among them are works by Edwards and Hewett (1993), Hightower and David (1991), and others.

McVean (2000) investigated use of different measurements of risk other than variance or semi-variance as was proposed by Ball and Savage (1999). He showed that efficient portfolios are highly dependent on the way risk is defined. That is, portfolios that were efficient with one definition of risk are not always efficient with another. Therefore, companies should assign priorities to their objectives and define risk and return in their analysis accordingly.

Allan (2003) considered differences in efficient portfolios obtained using different levels of future price. In his work, he examined only simple expected, high, and low price forecast structure, and he did not use the whole distribution of price forecasts.

Faulder and Moseley (2003) explained their "top-down" approach in applying portfolio theory to petroleum property investment, as opposed to industry used conventional "bottom-up" approach.

A number of other papers were written on this subject. The one that is worth mentioning is by Davidson and Davis (1995), where the authors illustrated simple and as they believe effective model for E&P portfolio optimization, which is not based on Markowitz's ideas.

2.3 Price Uncertainty Quantification

Oil and gas price forecast is an important part of any E&P economic model. According to Campbell *et al*. (2001), errors in project evaluation are more attributable to price uncertainty than any other uncertainty type. Therefore, a rigorous price risk quantification method is an essential part of any economic model. In our portfolio optimization model, we decided to use price forecasting based on sequential Gaussian simulation. According to conducted literature survey, due to relative novelty of the method nobody has done this before.

Sequential Gaussian Simulation (SGS) has been employed by geostatisticians to model spatial correlation in reservoir properties. Holmes *et al*. (2006) recently applied SGS to model temporal correlation in oil price data and used it to generate multiple equi-probable future price realizations for quantifying price uncertainty.

Olsen *et al*. (2005) investigated applicability of five different price uncertainty quantification models, such as conventional, bootstrap, inverted hockey stick, historical,

and SGS in petroleum project performance evaluation. They found that conventional hockey stick price forecasts typically underestimate the volatility of future oil and gas prices, often significantly, and recommended SGS as the most rigorous and at the same time most difficult to implement technique.

CHAPTER III

METHODOLOGY

3.1 Oil Price Forecasting Module

In their paper summarizing 5 oil price forecasting methodologies commonly used at the moment by industry practitioners, Olsen *et al.* (2005) pointed out that the method based on Sequential Gaussian Simulation (SGS) is the most rigorous and as a result, the most recommended one. Its major advantage over Bootstrapping method is that it takes into account both the frequency distribution and the semi-variogram of uninflated historical price data. The semi-variogram describes the historically observed temporal variability in the price of oil. Use of the semi-variogram ensures that changes in future oil price with time are consistent with historical changes leading to the most realistic forecasts. Therefore in our computer model, we decided to use this price forecasting methodology.

In order to implement the SGS method, we followed procedure described by Holmes *et al*. (2006) First we took the historical monthly average spot price data for West Texas Intermediate (WTI) starting from January 2000 until June 2007. This information is available in the tabular form from the Energy Information Administration web site (Energy Information Administration, 2007). Our price data is spread over considerable period of time (7.5 years). Therefore, inflation should be accounted for. Consumer Price Index (CPI) data for the same period of time has been taken from U.S. Department of Labor, Bureau of Labor Statistics web site (U.S. Department of Labor, 2007). Using CPI data, historical price data has been brought to common dollar values,

1st of July, 2007 U.S. dollar has been chosen as the common dollar (**Fig. 3.1**). Both actual historical prices, as well as inflation adjusted prices are shown on **Fig. 3.2**.
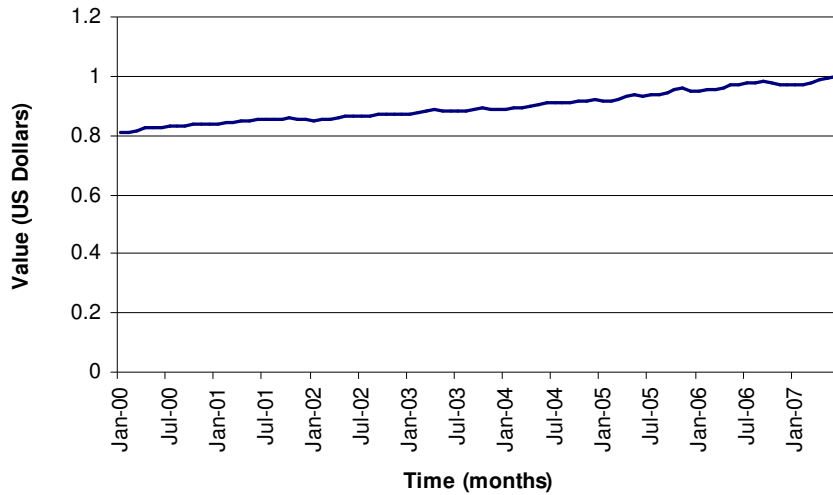
Fig. 3.1 - Value of July 1, 2007 US dollar
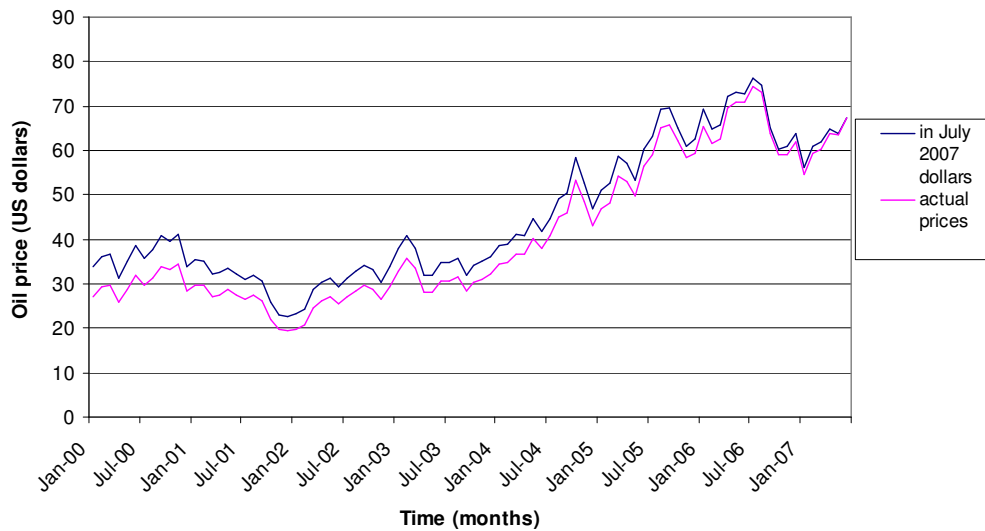
Fig. 3.2 - WTI actual historical prices and prices brought to

common July 2007 dollars

In order to use SGS, oil price data should be normally distributed. Distribution of our inflation adjusted price data is far from a normal distribution (**Fig.3.3**). Therefore we needed to apply a normal score transformation to our data. The normal score transformation ranks each data point and assigns it a corresponding position in the standard normal distribution. **Fig.3.4** shows inflation adjusted price data after being
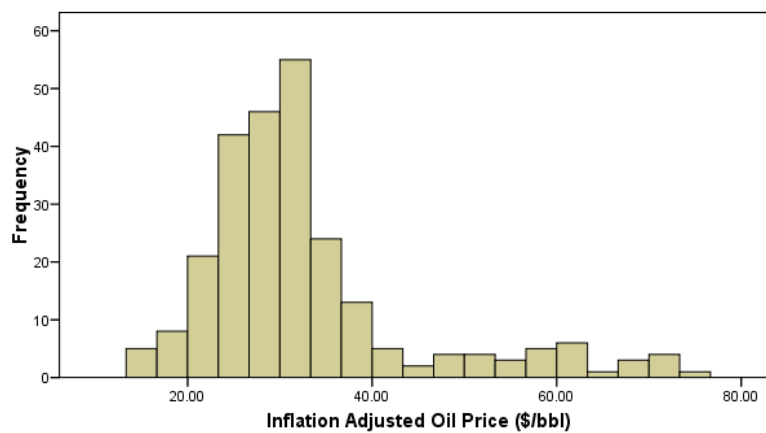
Fig. 3.3 - Histogram of inflation adjusted oil price data
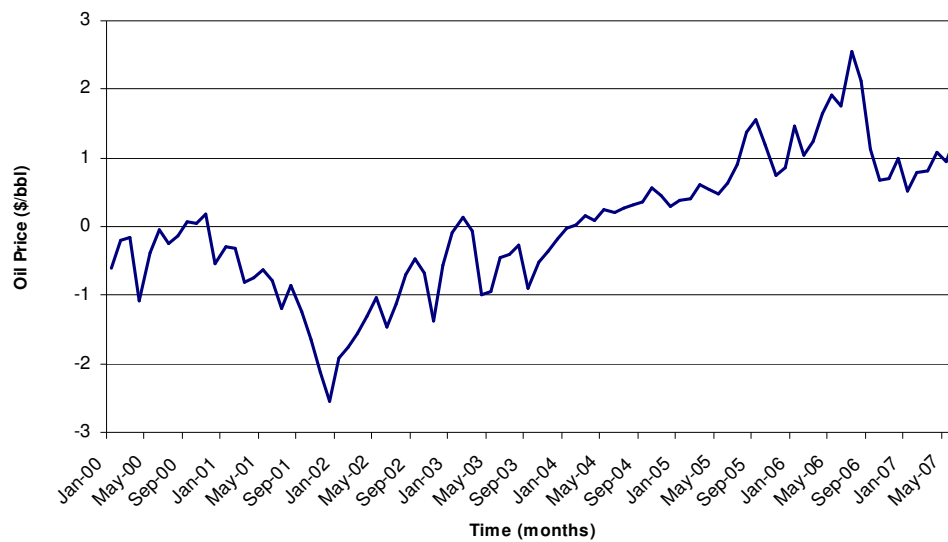
Fig. 3.4 – Normal score transformed oil price data

normal score transformed, while **Fig.3.5** shows its distribution. As it can be seen, now our data has standard normal distribution.
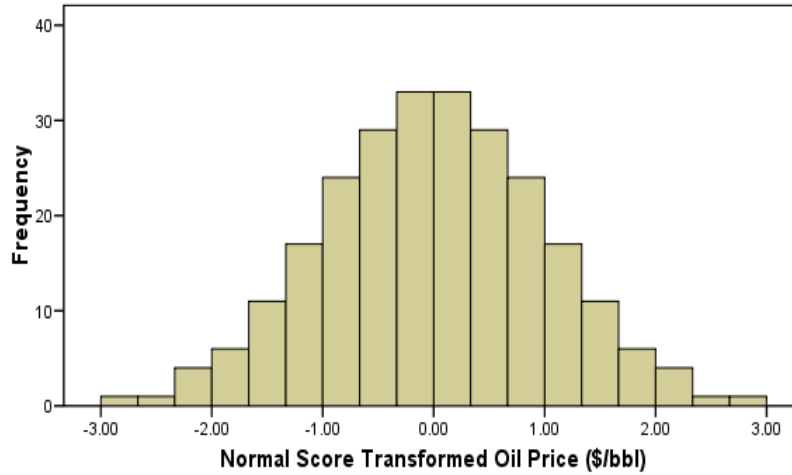
Fig. 3.5 – Histogram of normal score transformed oil price data

Now using this normally distributed price data, we may implement the temporal continuity model of oil price change in the form of a semivariogram. This semivariogram shows dependence of oil price variance to the lag-time. It is computed as follows. Let $z(t_a)$ represent the price value at time t_a, and $z(t_a + h)$ represent the price value h lag time units apart. All observations h time units apart from each other are included in the number of pairs, $N(h)$. Datum $z(u_a)$ is the tail and $z(u_a + h)$ is the head of the vector. Means and standard deviations of the head and tail of the vector may be calculated. The semivariogram is

$$\gamma^*(h) = \frac{1}{2N(h)} \sum_{a=1}^{N(h)} [z(t_a) - z(t_a + h)]^2 \qquad (3.1)$$

where $\gamma^*(h)$ is the estimated semivariogram for lag **h**. The semivariogram is half the average squared difference between the values of each pair (King, 2000). The semivariogram for our normal score transformed price data is shown on **Fig.3.6** by a dotted line. A solid line represents fit with the spherical model (Houlding, 2000). Equation of solid line is

$$\gamma(h) = \begin{cases} c_0 \left[\dfrac{3}{2} \dfrac{h}{a_0} - \dfrac{1}{2} \left(\dfrac{h}{a_0} \right)^3 \right], & \text{for } h \leq a_0 \\ c_0, & \text{for } h > a_0 \end{cases} \tag{3.2}$$

where $a_0 = 32$, and $c_0 = 1$.
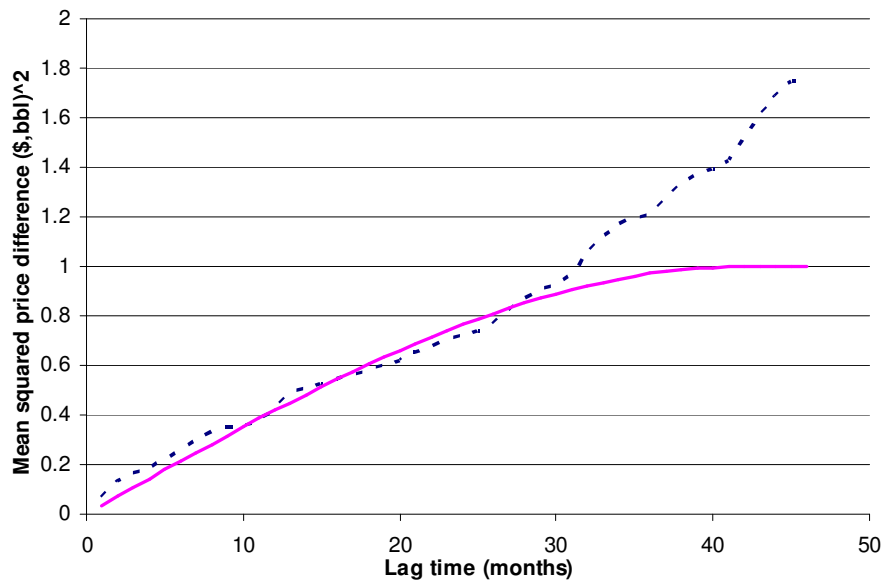
Fig. 3.6 – Semivariogram of normal score transformed oil price data

After fitting the real data semivariogram with the spherical model and having the equation of a spherical model, we can proceed with SGS. SGS is based on local neighborhood algorithm, which predicts unknown values by randomly sampling a

conditional probability distribution. In our case, the conditional probability distribution for a given month was determined by the mean and variance from ordinary kriging (OK) that uses price, either previously predicted or actual, in the preceding months. Randomly sampling this conditional distribution yields a predicted price for a given month. A complete oil price realization is created by predicting prices one month at a time, starting from July 2007, and moving forward sequentially. Since the conditional distribution is based on the historical data, the realization honors both the frequency distribution and the semivariogram of the historical data (Holmes *et al.* 2006).

The algorithm for SGS is as follows:

1. If the price data is not univariate normal, transform the data to obtain normal scored prices.

2. Construct a model of temporal continuity for the normal scored data.

3. Define a random path through all of the months to be simulated, that is each month exactly once.

4. Use Ordinary Kriging to determine the mean and variance of the Gaussian conditional probability distribution at a given month. Retain a specified number of neighboring data to be used as conditioning data. Both previously simulated price values and original data are included.

5. Draw randomly from conditional probability distribution and assign that value to the node being simulated.

6. Repeat steps 4 and 5 for all simulation months.

7. Back transform the simulated normal values into the original price values using original inflation adjusted price data distribution.

8. Repeat steps 3 – 7 for multiple realizations.

We believe step 4 requires some more explanation. Ordinary kriging is the technique developed in geostatistics to interpolate the value of an attribute at an unobserved location from observations of its value at nearby locations. It assigns weights to each available observation based on its distance from unobserved location using semivariogram, and calculates expected value of attribute at given unobserved location and its error by the following formulas (Kanevski & Maignan, 2004)

$$
\hat{Z}(x^*) = \begin{pmatrix} z_1 \\ M \\ z_n \\ 0 \end{pmatrix}' \begin{pmatrix} \gamma(x_1,x_1) & \Lambda & \gamma(x_1,x_n) & 1 \\ M & O & M & M \\ \gamma(x_n,x_1) & \Lambda & \gamma(x_n,x_n) & 1 \\ 1 & \Lambda & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} \gamma(x_1,x^*) \\ M \\ \gamma(x_n,x^*) \\ 1 \end{pmatrix} \tag{3.3}
$$

$$
\sigma^2_{OK} = \begin{pmatrix} \gamma(x_1,x^*) \\ M \\ \gamma(x_n,x^*) \\ 1 \end{pmatrix}' \begin{pmatrix} \gamma(x_1,x_1) & \Lambda & \gamma(x_1,x_n) & 1 \\ M & O & M & M \\ \gamma(x_n,x_1) & \Lambda & \gamma(x_n,x_n) & 1 \\ 1 & \Lambda & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} \gamma(x_1,x^*) \\ M \\ \gamma(x_n,x^*) \\ 1 \end{pmatrix} \tag{3.4}
$$

where z_1 … z_n are attribute values at location; x_1 … x_n, $\hat{Z}(x^*)$, expected attribute value at unobserved location x^*; σ^2_{OK}, interpolation error; $\gamma(x_1,x_n)$, semivariogram value at h equal to the distance between locations x_1 and x_n (see **Eq.3.1**)

In the case of price forecasting, attribute is the oil price, unobserved locations are months to be simulated, and observations are normal ranked historical monthly data. Though Holmes *et al*. used GSLIB© software to generate their forecasts, above
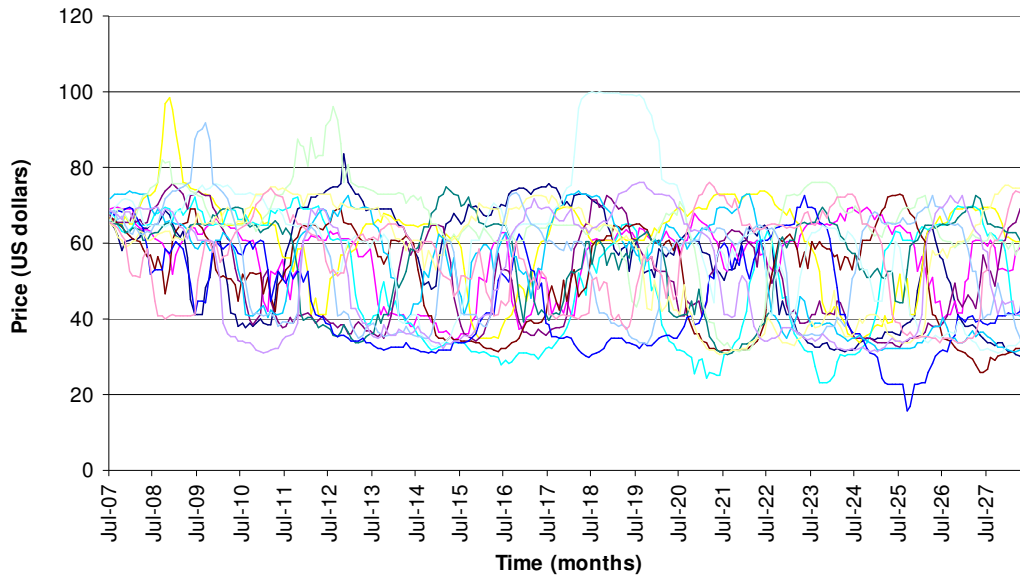
Fig. 3.7 – Uninflated SGS oil price forecasts
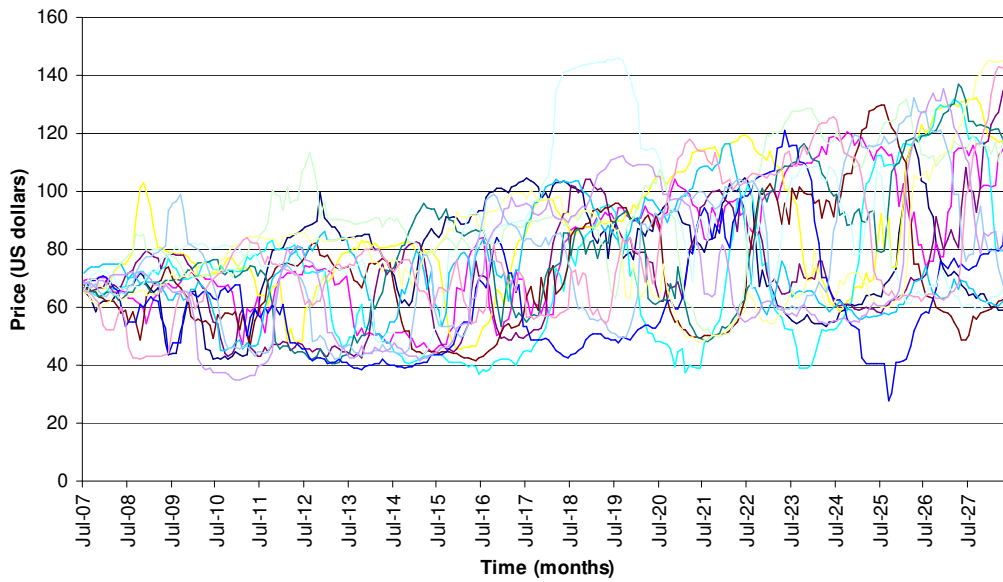
Fig. 3.8 –Inflated SGS oil price forecasts

described complex SGS algorithm has been implemented in MATLAB® computational environment. We have generated 50 equiprobable price forecast realizations starting

from July 2007 through June 2028. 12 of these realizations are depicted on **Fig.3.7**. However these forecasts are in July 2007 U.S. dollars, therefore annual inflation rate of 3% has been added to them, resulting in inflated price forecasts shown on **Fig.3.8**.

For further convenient access in the model all 50 realizations were arranged in an 252 x 50 array, with each realization being a column and each month a row. Thus we have completed price forecasting module of our model and proceeded to the next one.

3.2 Production Forecasting Module

Every E&P project's original oil in place figures (OOIP) were simulated using formula for volumetric estimates. Volumetric estimates of oil and gas reserves are generally used at early times in the life of a field. These estimates are considered preliminary as compared to the estimates obtained from using historical performance. The formula (Mian, 2002)

$$N = \frac{7758 * \phi * (1 - S_w) * h * A}{B_0} * R_E \qquad (3.5)$$

where N is reserves, STB; ϕ, porosity, fraction; S_w, formation water saturation, fraction; h, formation thickness, feet; A, drainage area, acres; B_0, formation volume factor, rb/STB; R_E, recovery efficiency, fraction.

We used Monte Carlo simulation to obtain full distribution of reserves of the given field. Each of the parameters in the volumetric formula was distributed according to its naturally occurring distribution shape, taking into account correlations between

different parameters. Distribution shapes for each parameter (Cronquist, 2001) are described in **Table 3.1**.

Table 3.1 Naturally occurring distribution types of volumetric analysis parameters

Parameter	Type of Frequency Distribution	Correlations
Porosity, ϕ	Typically Gaussian, but may exhibit slight skew, either positive or negative; tendency towards positive skew and log-normal distributions in carbonates and negatively skewed beta distributions in sandstones. Typically exhibits covariance with initial water saturation, S_w.	
Interstitial water saturation, S_w	Typically exhibits negative skew, but may approach a symmetric distribution, or a distribution with a positive skew.	
Net pay, h	Typically exhibits positive skew, with log-normal being a good approximation. May exhibit covariance with drainage area.	
Permeability, k	Typically exhibits positive skew, with log-normal being a good approximation; nature of the distribution may be controlled by the type of depositional environment and/or post depositional diagenesis; typically, covariant with porosity.	
OOIP, N	Generally exhibits positive skew, with log-normal being a good approximation.	
Recovery efficiency, R_E	Typically exhibits positive skew, with log-normal being a good approximation; may exhibit covariance with porosity, irreducible water saturation, permeability, and/or net pay, depending on drive mechanism.	
Initial well potential	Typically exhibits positive skew, with log-normal being a good approximation.	
Drainage area, A	May be well approximated by Gaussian normal distribution. Typically, covariant with net pay.	

Fig. 3.9 shows simulated physical parameters data that was generated using MATLAB® for application in the model. In this figure recovery coefficient R_E

distributed log-normally is correlated with the normally distributed porosity, with correlation coefficient, $\rho = 0.6$. In the same manner, net pay, h distributed log-normally is covariant with the drainage area, distributed normally. Our model has been developed in such a way, allowing not only paired correlations but also triple, quaternary, and so on
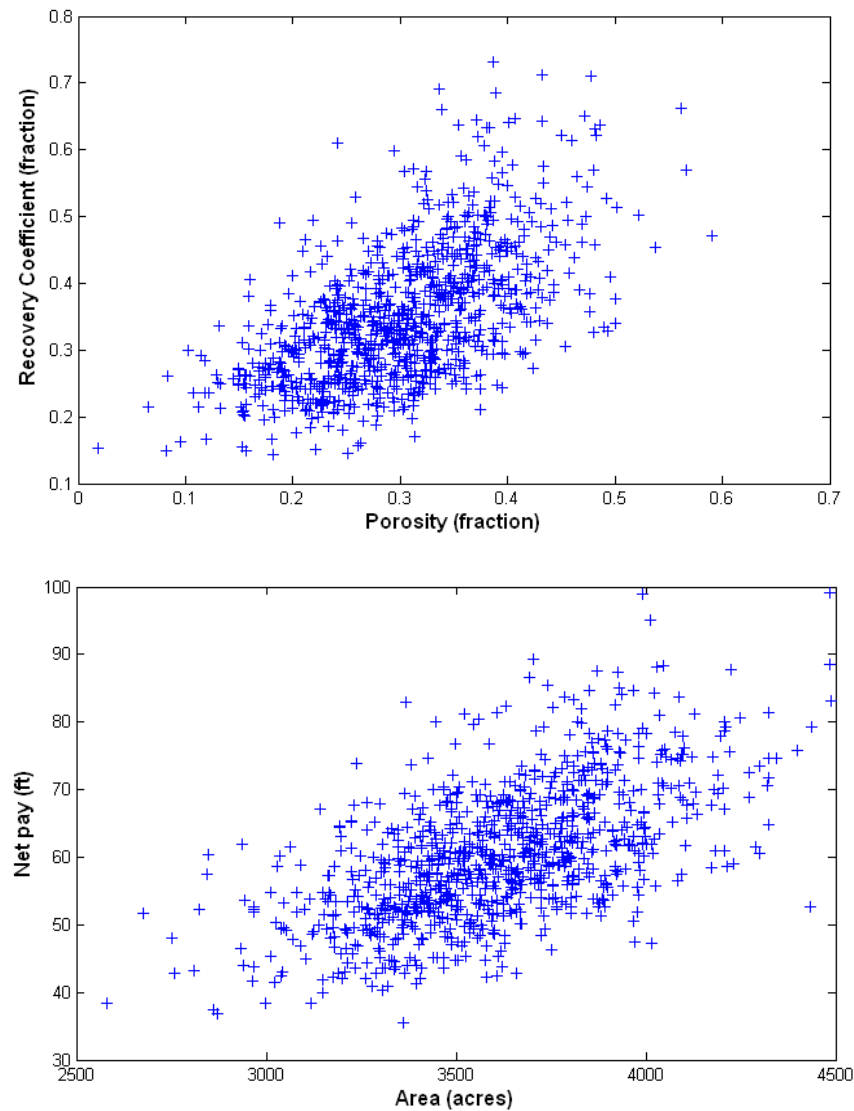
Fig. 3.9 - Correlation between different parameters in the

volumetric estimates formula

up until n[th] level of correlation, with different correlation coefficient values. For example, the same porosity numbers were correlated both with interstitial water saturation and with recovery efficiency coefficient, which led to the secondary correlation between latter two parameters. Moreover, correlated parameters may have different shapes of distributions, even individually specific, defined by histogram.

Monte Carlo simulation with n = 1000 iterations has produced following distribution of reserves for given filed, showed at the **Fig. 3.10**.
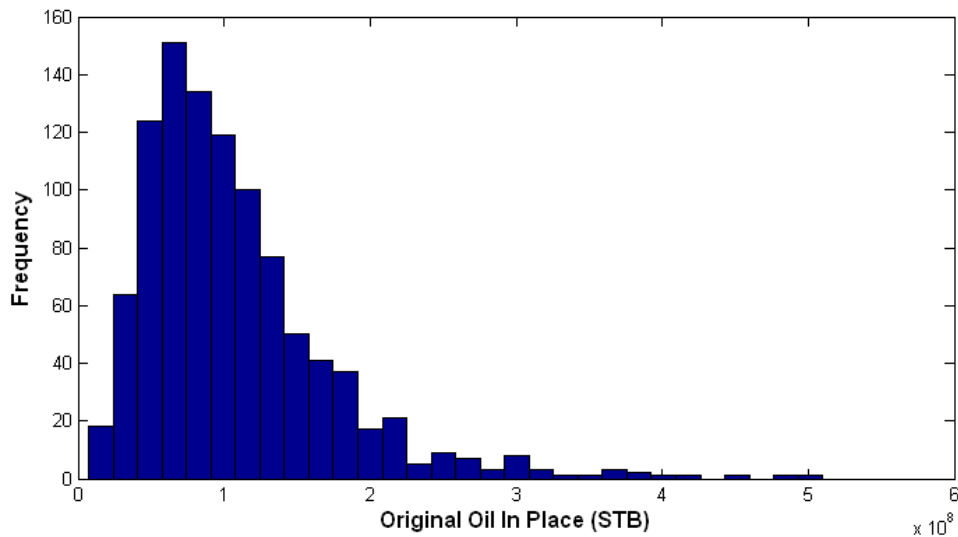
Fig. 3.10 - Distribution of OOIP, n =1000

As you can see, distribution is positively skewed, well resembling log-normal distribution. This is an expected result, as it is shown in **Table 3.1**, OOIP figures usually in nature are distributed log-normally. This can be explained by the statistical *Central Limit Theorem*. According to this theorem, product of random variables is the random variable itself with distribution approaching log-normal distribution as the number of

random variables approaches infinity, independent of the distribution shapes of random variables (Montgomery, 2007).

$$P = \lim_{n \to \infty} \prod_{i=1}^{n} X_i \qquad (3.6)$$

where $P \sim$ log-N(μ, σ^2); $X_i \sim$ any distribution.

Since the original oil in place numbers calculated from **Eq.3.5** is the product of specifically distributed random variables, it is reasonable to expect them to be log-normally distributed: the result that we got in our model.

The next step was to come up with forecasted production profiles. For this purpose, we used the hyperbolic decline curve formula proposed by Arps (Seba, 1998) (**Eq.3.7**). The hyperbolic decline curve is a concave upward curve when plotted on semi logarithmic graph paper. As a consequence, the decline rate, *a*, is not a constant value but rather is the slope of the tangent to the rate-time curve at any point. The curvature of the curve is defined by hyperbolic exponent, *b*. Hyperbolic exponent is constant in time.

$$q_{ot} = q_i * (1 + b * a_i * t_0)^{-\frac{1}{b}} \qquad (3.7)$$

where q_{ot}, production rate at time t_0, STB/month; q_i, initial production rate, STB/month; a_i, initial decline rate, fraction; b, hyperbolic exponent.

In normal petroleum operations, the value of *b* ranges between 0 and 1.0. Value of b depends on fluid type, drive mechanism, formation type, whether stimulation has been applied, and number of other factors. Typical *b* values and situations when they can be expected (Fetkovich, 1994) are listed in the **Table 3.2**.

Fig. 3.11 shows frequency distribution of b hyperbolic exponent values as they occur in nature (Arps, 1944). As it can be seen from this figure, the majority of reservoirs have b value less than 0.5. Also, b values more than 1.0 are exhibited by tight gas/oil reservoirs and reservoirs that are still in infinitely acting, transitional flow. Thus by changing b values, as the rough, first level approximation, we may simulate different fluid types, formation types, and drive mechanisms. In our model initial production rate, q_i in **Eq.3.7** has been calculated as the percentage of total reserves from **Eq.3.5**. Value of b has been distributed according to the drive mechanism we wanted to simulate ($b=0$ for volumetric reservoir, $b \sim U(0.2, 0.4)$ for solution gas drive). Initial decline rate (a_i) has been calculated by equating ultimate production volume (N_p) in **Eq.3.8** (Seba, 1998) to previously calculated reserves volume from equation for volumetric estimates, and then

Table 3.2 Typical values of b and situations when they are observed

Value of b	Specific Situation
b indeterminable	• Constant or increasing rate • Flow all transient
$b = 0$, exponential	• Single phase liquid flow • High pressure gas • Depletion or solution gas drive with unfavorable k_g/k_o • Poor waterflood performance • Wells with high backpressure • Liquid loaded gas wells • Gravity drainage with no free surface
$b = 0.3$	• Typical for solution-gas drive
$b = 0.4$ to 0.5	• Typical for gas wells
$b = 0.5$	• Gravity drainage with a free surface • Full water drive in oil reservoir
$b > 0.5$	• Layered, no cross flow reservoirs
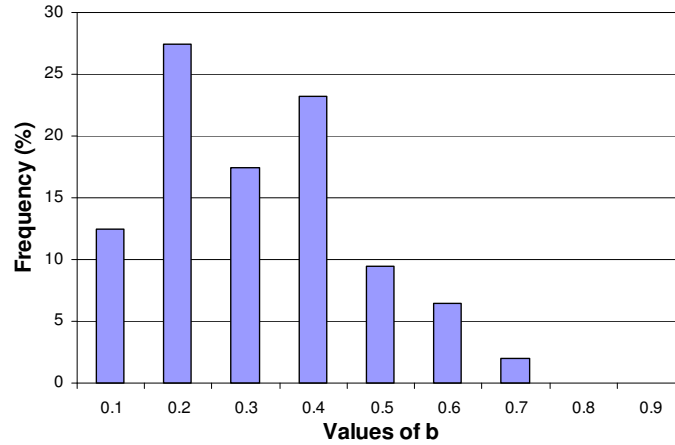
Fig. 3.11 - Distribution of hyperbolic exponent *b* in nature

solving **Eq.3.8** for a_i, with economic limit q_{EL} being equal to 1000 STB/month. This way, the model assures total depletion of previously estimated reserves before reaching economic limit, and no production in excess of estimated reserves.

$$N_p = \frac{q_i^b}{a_i(1-b)}\left[q_i^{1-b} - q_{EL}^{1-b}\right] \qquad (3.8)$$

Moreover, to come up with actual production profiles three more time-variables were introduced in the model: (1) length of delay period, when production rate equals 0; (2) length of development period, when production rate rises from 0 up until q_i; (3) length of production plateau period, when production stays at the level of q_i. After these three periods, production starts to decline according to **Eq.3.7** until it depletes all reserves.

All three time-period variables are distributed uniformly with the possibility of changing min and max according to the specific situation and expectations of the project analysts.

In our implementation, we considered four different production profile types, obtained by changing time-period variable values: (1) Accelerated profile, with minimum delay period, fast development period, and long plateau period; (2) Delayed profile, with considerable delay period, fast development, and long plateau; (3) Centered plateau profile, with intermediate delay and development periods, short plateau, and long subsequent decline; and (4) Profile with subsequent enhanced oil recover (EOR). **Figs. 3.12 – 3.15** show these profile types. First graph of each figure shows n=100 realizations of each profile type, while second graph shows generic shape of profile, by averaging all realizations at the first graph for each month. Let us remind that height of each realization curve (q_i) has been determined as percentage of total reserves, distribution of which is shown on **Fig. 3.10**.
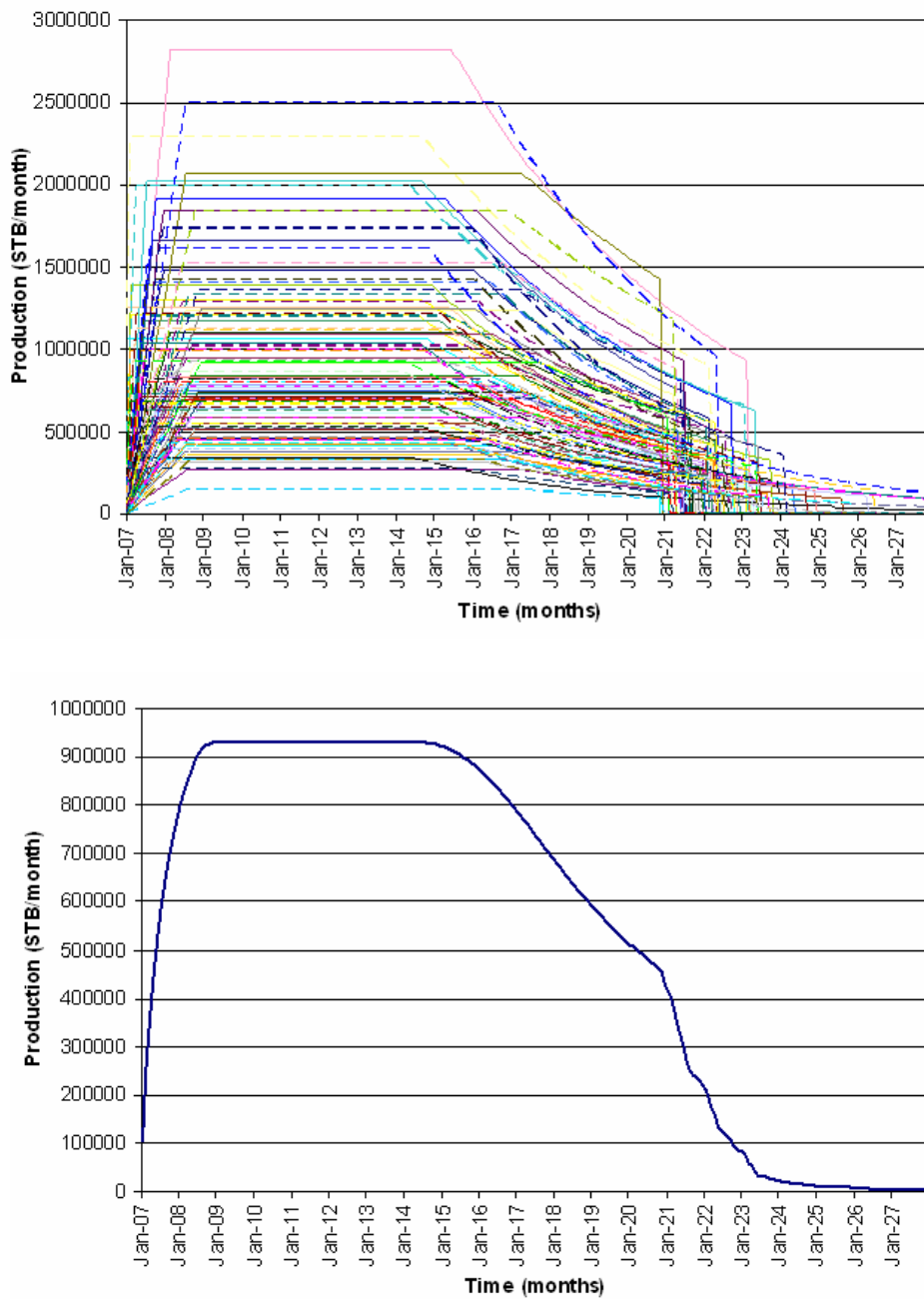
Fig. 3.12 - n =100 realizations of accelerated profile type and

average profile curve shape
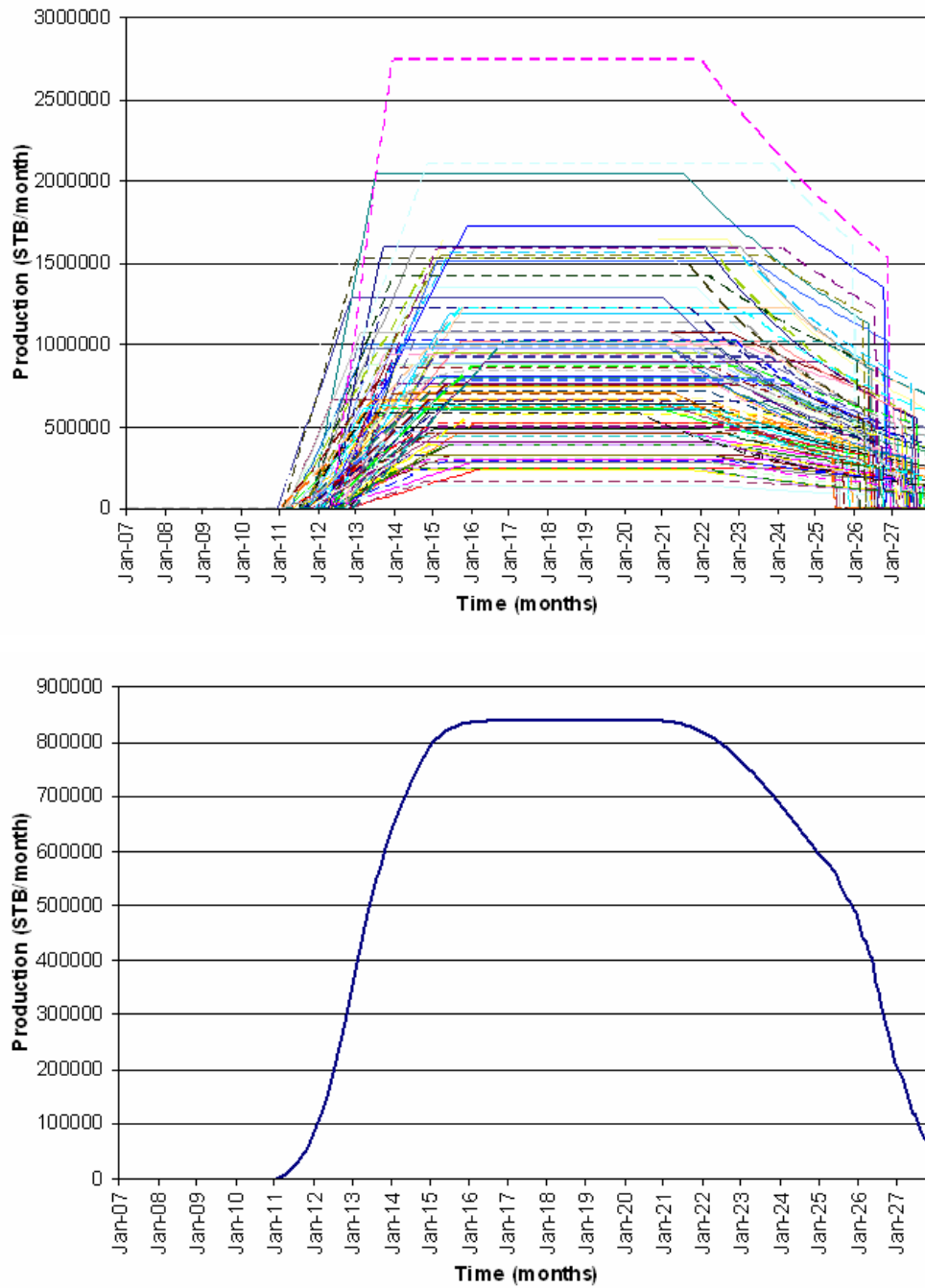
Fig. 3.13 - n =100 realizations of delayed profile type and

average profile curve shape

Fig. 3.14 - n =100 realizations of centered plateau profile type and

average profile curve shape

Fig. 3.15 - n =100 realizations of EOR profile type and

average profile curve shape

Minimum and maximum values of time-variables for production profiles shown on **Fig. 3.12 - 3.15** are summarized in the **Table 3.3**. As it has been said, these values can be easily changed by the users of the model, allowing them to come up with their own situation specific profile shapes, based on their best estimates.
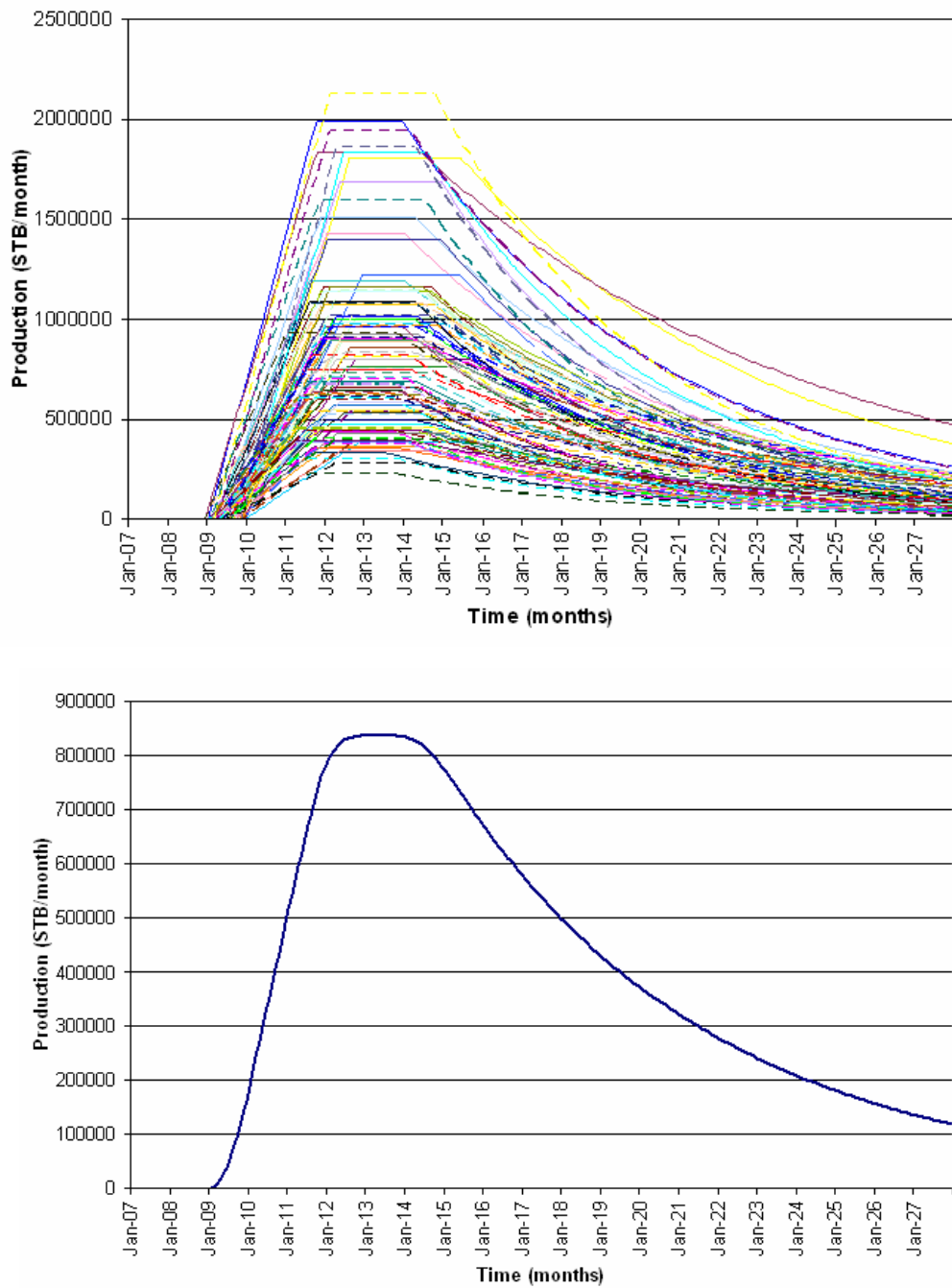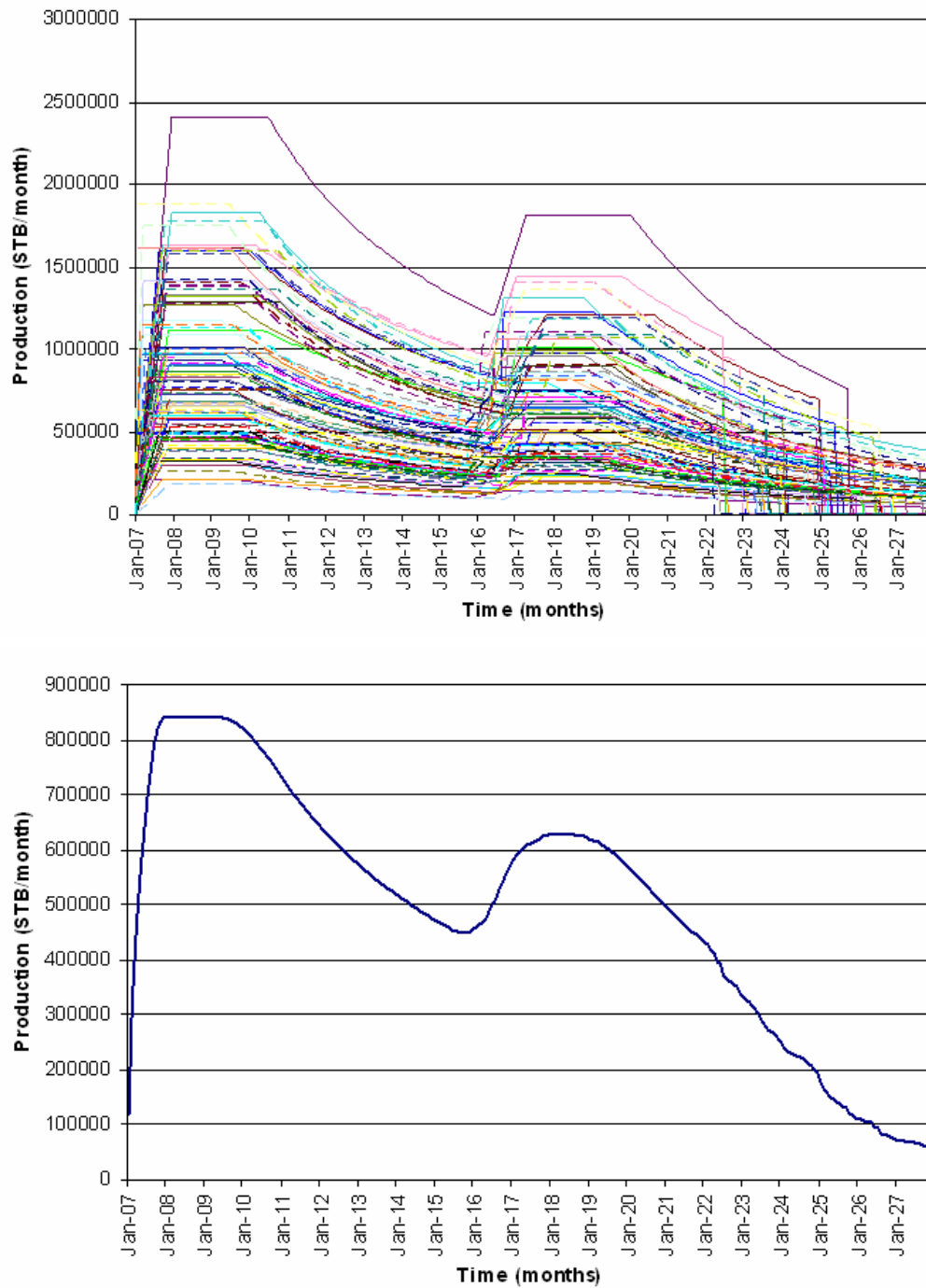
Table 3.3 Minimum and maximum values of time-variables for production profiles

Profile type	Delay period, months			Development period, months			Production plateau period, months		
	Min	Max	Distribution	Min	Max	Distribution	Min	Max	Distribution
Accelerated	0	2	uniform	0	24	uniform	84	108	uniform
Delayed	48	73	uniform	12	48	uniform	84	108	uniform
Centered	24	36	uniform	24	36	uniform	24	36	uniform
EOR	production profile is more complex								

In summarizing the work done for implementation of production forecasting module of our model, we would like to note that these production forecasts should be considered as the rough, first level approximations of actual production rates. During actual application of computer model for real life cases, we recommend using production forecasts from more reliable sources than simple decline curve analysis, such as history matched reservoir simulation.

3.3 Cash Flow and Performance Evaluation Module

Results of price forecasting and production forecasting modules were used to generate each project's cash flow. Based on this cash flow, each project's performance indicators were calculated.

Fig.3.16 shows typical E&P project cash flow scheme used in our model. The project timeline is divided on three major periods: exploration, development, and production. The project's entire cash outflows are categorized on two broad categories: capitalized and afterwards depreciated capital expenditures (CAPEX), and immediately expensed as occurred operating expenses (OPEX), whereas project's cash inflows consist of only one category: revenues from sale of oil. There are four major capital expenditures for simplicity denoted as CAPEX 1, 2, 3, and 4 (see **Fig.3.16**).
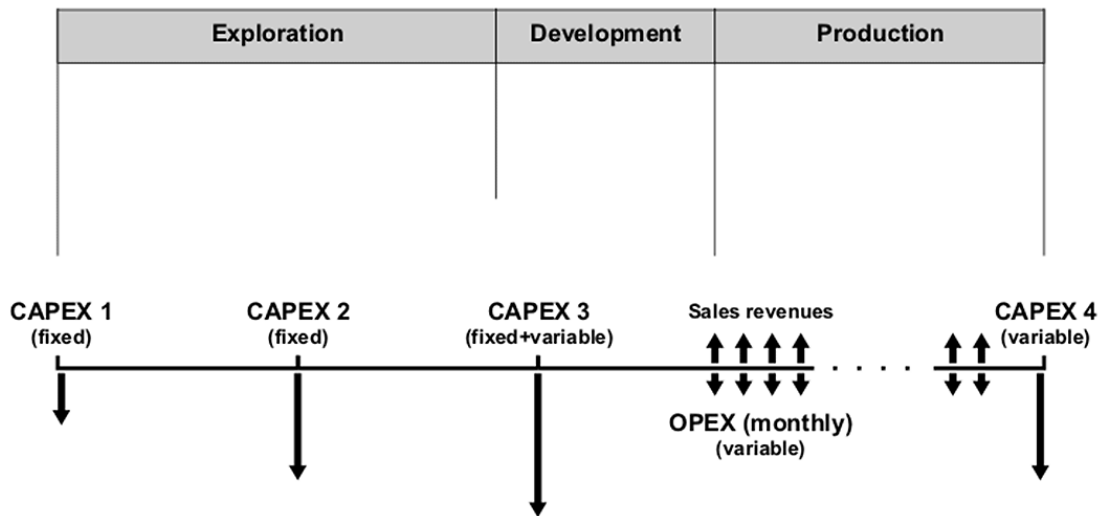
Fig. 3.16 - Typical E&P project cash flow scheme used in the model

The algorithm of the cash flow module works as follows. At this stage, the program already has the whole probability distribution of reserves, shown on the **Fig.3.10**. Corresponding to each of n=1000 realizations of reserves, program has generated n=1000 production profiles. Some of these profiles are economically viable, some are not. Based on each of these production profiles, the model generates a cash

flow profile. Then discounting this profile back to present, we come up with the value of economic indicator (NPV, PI, GRR) that shows whether this profile is economically successful or not. Thus every realization of reserves corresponds to one value of economic indicator. Consequently we get distribution of economic indicators, which is calculated based on the distribution of reserves, and this distribution also includes economically unfavorable outcomes.

To come up with a cash flow profile, first for each production profile model assigns fixed expenditure on G&G survey (CAPEX1). Next, after having results of the seismic survey and passage of some user defined period of time program assign fixed exploratory drilling expense (CAPEX2) to every of n=1000 realizations. After data from the exploratory wells have been acquired, we proceed with development and in all n=1000 realizations incur CAPEX3, field development expenses. These are the highest capital expenditures; therefore they consist of fixed part plus variable part, which vary with the volume of reserves to be produced. All these three initial capital outflows are divided by time periods that may be specified by the user of the model.

The logic behind this algorithm is as follows. At the time of field development we do not know with absolute certainty, whether the project eventually will yield positive net present value (NPV). Even after drilling production wells, there is a possibility of reserves being low resulting in losses. As will be seen later, the described approach yields distribution of major economic indicators which also include the probability of unfavorable results (NPV < 0, PI < 1, and GRR < hurdle rate), thus ensuring coverage of all possible outcomes.

In our illustration, all CAPEX and OPEX amounts were calculated so that mean of profitability index (PI) distribution would lay around 2.0. This would give us a whole distribution of possible outcomes including highly successful (PI > 2.0) as well as highly unsuccessful (0 < PI < 1.0).

At the stage of production, the project incurs only variable field operating expenses (OPEX) that vary with the production rate, and obtains cash inflows in the from of revenues from sales, which are calculated by multiplying production in given month with the price value at that month. Finally, after all reserves are depleted, the project incurs CAPEX4, abandonment costs, which are variable to the amount of ultimate production.

All three initial capital expenditure amounts are depreciated using straight line method. In our model, we have implemented full cost accounting procedures (Gallun, 2001), therefore, CAPEX1 and CAPEX2 are capitalized. Tax benefits from incurring losses are also accounted for in form of deferred tax assets. Corporate tax rate is 36 %. **Fig.3.17** shows the cumulative after tax net cash flow curves for centered plateau production profile type depicted on the **Fig.3.14**. As it can be seen, some of the curves do not go high enough and stay just above zero, thus not ever paying back and yielding negative NPV.

Using the after tax net cash flow data, we calculated project performance indicators for each iteration. Among many different types of performance indicators and return measures, we decided to stop on the following three: net present value, growth rate of return, and profitability index.

The net present value (NPV), also referred to as the present value of cash surplus or present worth, is obtained by subtracting the present value of periodic cash outflows from the periodic value of periodic cash inflows. The present value is calculated using the weighted average cost of capital of the investor, also referred to as hurdle rate (Mian, 2002). NPV is calculated using **Eq.3.9**.
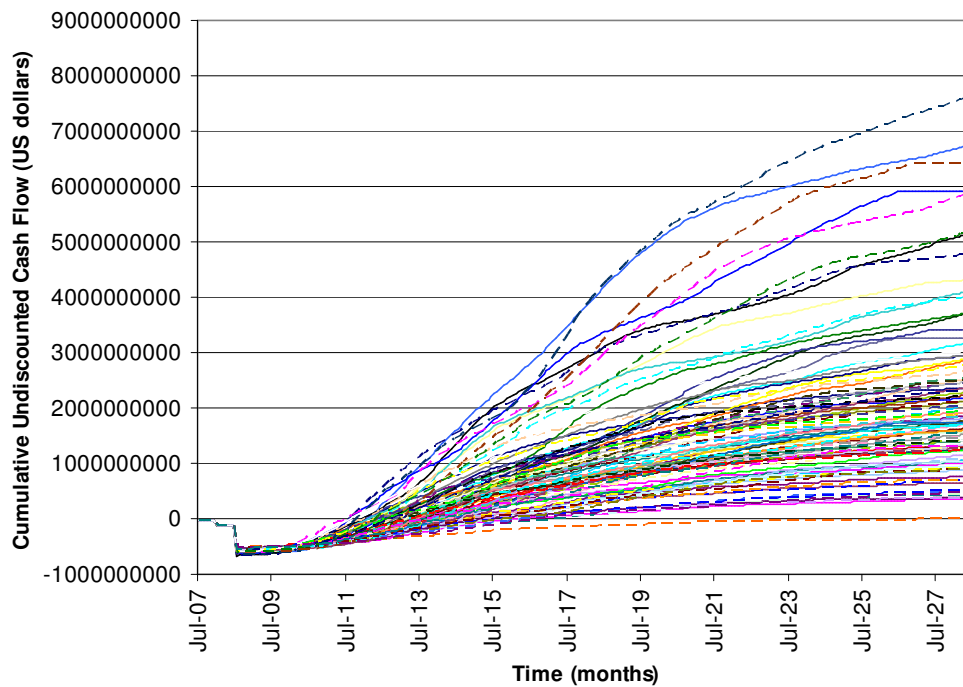
Fig. 3.17 - Cumulative after tax net cash flow curves

$$NPV = \sum_{t=1}^{n} \frac{NCF_t}{(1+i_d)^t} \qquad (3.9)$$

where NCF_t is after tax net cash flow at time t; i_d, the discount rate, i.e. the required minimum annual rate of return on new investment; n, project's economic life in units of t. The major reason for selecting NPV as one of the return measures for our model is that

its units of measurement are in present U.S. dollars. Thus NPV explicitly illustrates monetary increase in company's value brought by acceptance of the project.

We calculated NPV value at 12% discount rate for each after-tax net cash flow curve on **Fig.3.17**, thus obtaining NPV distribution illustrated on **Fig.3.18**. As it can be seen, shape of distribution is positively skewed, which is expected result, taking into account shape of reserves distribution on **Fig.3.10**. Moreover, NPV distribution also includes some probability of NPV being less than zero.
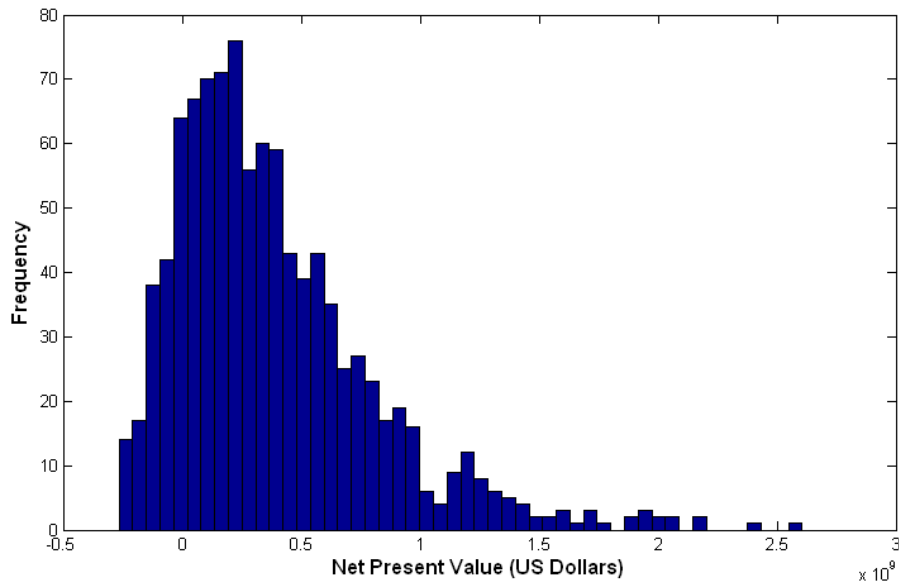
Fig. 3.18 - Distribution of NPV calculated at 12% discount rate

The growth rate of return (GRR) is calculated by first compounding all the positive net cash flows forward to some time horizon, *t* years in the future. Any cash flow beyond that time is discounted back to this point. Secondly, all the negative cash flows discounted to time zero to get the present value of investment. The rate at which the positive cash flows are compounded and the negative investments are discounted is

the opportunity rate of the organization. GRR is the rate at which present value of negative investments should grow to yield future value of all positive net cash flows at time t. GRR is calculated using **Eq.3.10**.

$$GRR = \left(\frac{FV_t}{PV_0}\right)^{1/t} - 1 \qquad (3.10)$$

where FV_t, future value of all positive net cash flows at time t; PV_0, present value of all negative net cash flows at time zero.

GRR is the measure of percentage return. At the same time, it does not have such shortcomings as possibility of multiple rates, reinvestment rate assumption, and trial-and-error calculation that internal rate of return, alternative measure of percentage return, has. Therefore we decided to include it in our analysis. Distribution of GRR calculated based on **Fig.3.17** data is shown on **Fig.3.19**. Here again, some area of the
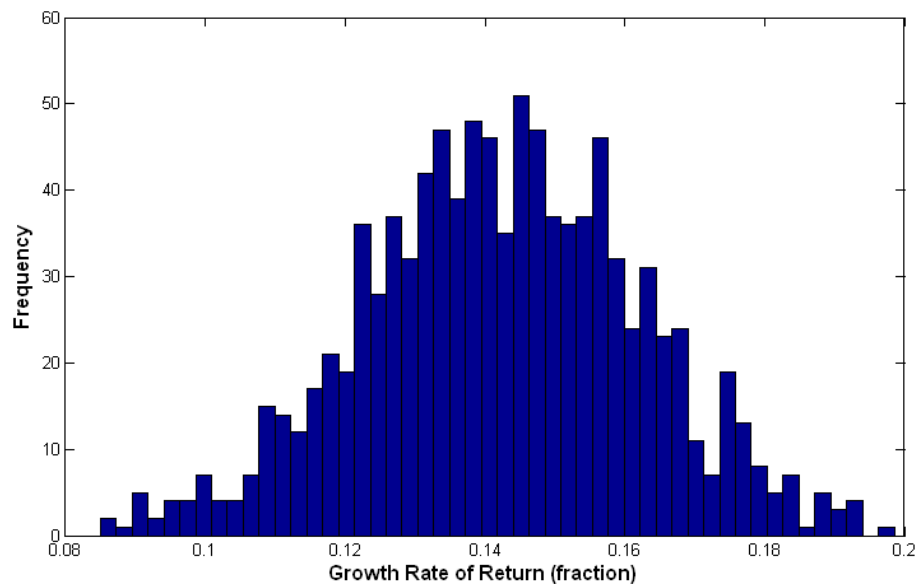
Fig. 3.19 - Distribution of GRR calculated at 12% discount rate, t=25 years

histogram lies below 12% hurdle rate, indicating probability of losses.

The third return measure used in our model, profitability index (PI), is a unit less performance indicator that simply shows how much present value benefits created per dollar of investment when company accepts the project. Formula for PI calculation is shown next.

$$PI = 1 + \frac{NPV}{PV_0} \qquad (3.11)$$
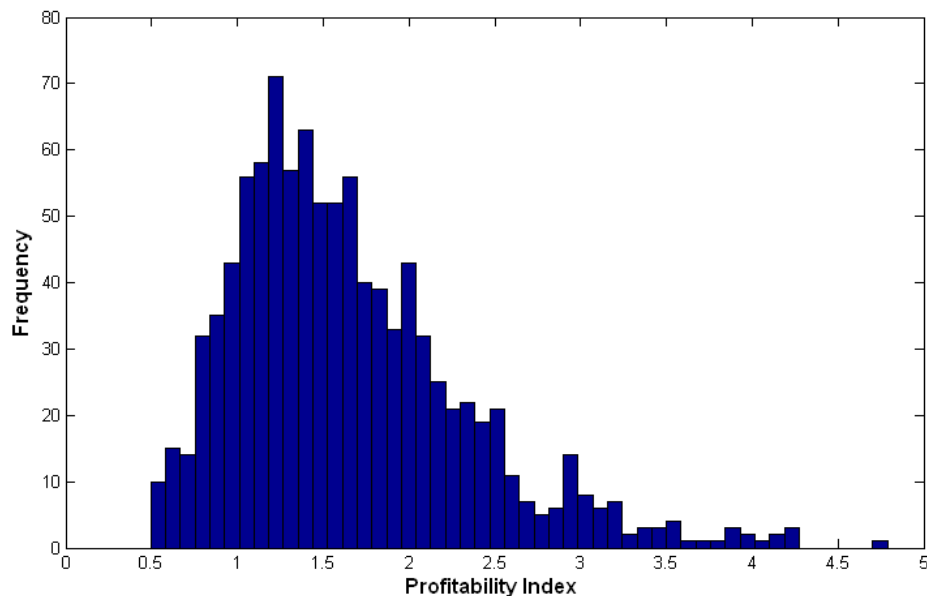
Fig. 3.20 - Distribution of PI calculated at 12% discount rate

Fig.3.20 shows distribution of PI calculated for each net cash flow curve shown on the **Fig.3.17**. As you can see, the distribution is positively skewed, with log-normal being a good approximation.

Thus, we have implemented the procedure of obtaining full distribution of return for any individual petroleum asset. Moreover, we have obtained full distributions for

three different measures of return. Now, we may proceed with using these distributions to optimize a portfolio that consists of a number of individual assets.

3.4 Portfolio Optimization

Table 3.4 shows major variables of production forecasting and cash flow handling modules, and the way these variables may be changed by the user of the model. By means of changing these variables and applying four different production profile types, explained in the **section 3.2**, we have synthesized eleven different template projects that may be included in company's portfolio. **Table 3.5** describes every project's distinction from the others.

In our illustration we have differentiated projects based on the type of drive mechanism and expected production profile shape. We believe that both these factors are important causes of differences between projects, though we also realize that there are some other important differences such as whether reserves are short or long, geographic location of reserves, proximity to markets, etc. We also assumed that both drive mechanism and expected production profile shape would be beforehand determined by reservoir engineers and available for decision making.

For each of eleven synthesized projects, the full distribution of PI, GRR, and NPV data has been generated in the way it is explained above. These return distribution parameters are shown in **Table 3.6**. We applied our portfolio optimization model to this set of synthesized projects to test its functionality and obtain optimized portfolio.

Table 3.4 Major variables of production forecasting and

cash flow handling modules

Variable	Options that may be changed
Number of iterations, n	Single value
Area, A	Distribution shape and parameters
Net pay, h	
Water saturation, S_w	
Porosity, φ	
Recovery factor, R_E	
Formation volume factor, B_0	
Initial maximum production rate, q_i	Single value percentage of N
Hyperbolic exponent, b	Distribution shape and parameters
Delay time period length @ q=0	
Production increase period length	
Plateau period length	Distribution shape and parameters
Production profile shape	Complex curve specified by the user
Percentile to be rejected after G&G	Single value
Percentile to be rejected after exploratory drilling (dry hole rate)	Single value
G&G costs	
Exploratory drilling costs	
Development costs, fixed component	
Development costs, variable component	
Operating expenses/STB produced (lifting costs)	Single value
Tax rate	Single value percentage
Weighted average cost of capital, discount rate	Single value

Table 3.5 Description of eleven synthesized projects

Project Code	Project Description
ACC	Accelerated profile, volumetric reservoir
ACC_SOL	Accelerated profile, solution-gas drive
DLD	Delayed profile, volumetric reservoir
DLD_SOL	Delayed profile, solution-gas drive
DBL	EOR profile (with double plateaus), volumetric reservoir
DBL_SOL	EOR profile (with double plateaus), solution-gas drive
CNT	Centered plateau profile, volumetric reservoir
CNT_SOL	Centered plateau profile, solution-gas drive
TGH	Tight oil reservoir (*b* distributed ~ 2.0 – 5.0)
HRP	So-called high return project
LRP	So-called low return project

Table 3.6 Return distribution parameters for eleven synthesized projects

Project code	Mean of PI	Standard deviation of PI	Mean of GRR, fraction	Standard deviation of GRR	Mean of NPV, $	Standard deviation of NPV
ACC	2.7068	1.1555	0.1663	0.0212	1.22E+09	9.61E+08
ACC_SOL	3.5535	1.4066	0.1801	0.0201	1.78E+09	1.26E+09
DLD	5.3770	1.2627	0.2027	0.0128	1.11E+09	7.02E+08
DLD_SOL	5.6929	1.2819	0.2057	0.0117	1.17E+09	7.54E+08
DBL	3.7317	1.4229	0.1828	0.0192	1.72E+09	1.22E+09
DBL_SOL	3.4682	1.3019	0.1792	0.0196	1.63E+09	1.22E+09
CNT	2.4235	0.8232	0.1623	0.0175	9.04E+08	6.95E+08
CNT_SOL	2.5768	0.9127	0.1651	0.0182	9.41E+08	6.85E+08
TGH	3.7626	1.1318	0.1844	0.0158	2.24E+09	1.42E+09
HRP	6.8289	1.3792	0.2149	0.0108	2.46E+10	1.34E+10
LRP	1.6387	0.6526	0.1423	0.0191	3.81E+08	4.08E+08

An important point to note here is that while distributions of reserves may be absolutely independent of each other (e.g. they represent geographically distant fields), the distribution of their economic indicators should actually indicate some dependence, because all fields are producing in the same oil price environment. In our model, this fact was realized in the following way. First we numbered each of 50 equiprobable price forecasts starting from 1 to 50. Then we generated a 1000x1 dimensional vector of random numbers from 1 to 50 and used this vector as the order of forecasts. Then we applied forecasts to every of n=1000 production profiles according to this order, without changing order when moving from one project to another. In this way we ensured that price forecast, e.g. number 26 was applied to the 1st profile (out of 1000 available) of every of 11 projects, forecast number 42 was applied to the 2nd profile of every of 11 projects, and so on.

In matrix form, the portfolio optimization problem can be described as follows. Consider an investor that seeks a best allocation of wealth among a basket of risky assets, called a portfolio. The best can be defined as an allocation such that the risk incurred is minimum for that level of expected return or the expected return is maximum for that level of risk. The data of the problem consists in an array of returns (Dias, 2003), where each component i of this array is the expected return to the asset i in the considered horizon:

$$r = (r_1, r_2, \ldots, r_n)$$

We have also a covariance matrix as the shown below:

$$Cov = \begin{bmatrix} \sigma_1^2 & \sigma_{21} & \Lambda & \sigma_{n1} \\ \sigma_{12} & \sigma_2^2 & \Lambda & \\ M & M & O & \\ \sigma_{1n} & \sigma_{2n} & \Lambda & \sigma_n^2 \end{bmatrix}$$

where σ_i^2 is variance of asset i's return; σ_{ij}, covariance between assets i and j.

This matrix is positive semidefinite. The expected return for a portfolio with n assets is

$$R_p = x'r \qquad (3.12)$$

where each component i of the array $x = (x_1, x_2, ..., x_n)$ is the fraction of the investor's total wealth allocated in the asset i. The portfolio risk (variance of return) is

$$\sigma_p^2 = x'Cov\ x \qquad (3.13)$$

By assumption, the investor will allocate all his wealth in the selected portfolio.

Having return distributions of the assets in consideration, we constructed such covariance matrixes between assets. An example covariance matrix for PI of 11 synthesized assets is shown in **Table 3.7**. As you can see, covariances between all 11 projects are positive, indicating positive correlations between returns of projects. This is explained by common price environment for all 11 projects, which was implemented in the way explained above.

Table 3.7 Return covariance matrix for PI distribution of eleven synthesized projects

	ACC	ACC_SOL	DLD	DLD_SOL	DBL	DBL_SOL	CNT	CNT_SOL	TGH	HRP	LRP
ACC	1.3351	0.1123	0.0723	0.1601	0.2202	0.1656	0.0246	0.0848	0.0930	0.1575	0.0148
ACC_SOL	0.1123	1.9784	0.2335	0.2440	0.1098	0.1521	0.0218	0.0577	0.1262	0.3397	0.0287
DLD	0.0723	0.2335	1.5944	0.5089	0.2168	0.2595	0.1540	0.2251	0.2503	0.4641	0.0947
DLD_SOL	0.1601	0.2440	0.5089	1.6433	0.2673	0.2410	0.1750	0.2246	0.3202	0.4991	0.1770
DBL	0.2202	0.1098	0.2168	0.2673	2.0247	0.1357	0.0956	0.1614	0.1110	0.3590	0.0516
DBL_SOL	0.1656	0.1521	0.2595	0.2410	0.1357	1.6949	0.1263	0.0748	0.2251	0.2248	0.0564
CNT	0.0246	0.0218	0.1540	0.1750	0.0956	0.1263	0.6776	0.1139	0.1001	0.2399	0.0267
CNT_SOL	0.0848	0.0577	0.2251	0.2246	0.1614	0.0748	0.1139	0.8330	0.1065	0.2423	0.0502
TGH	0.0930	0.1262	0.2503	0.3202	0.1110	0.2251	0.1001	0.1065	1.2809	0.3635	0.0758
HRP	0.1575	0.3397	0.4641	0.4991	0.3590	0.2248	0.2399	0.2423	0.3635	1.9023	0.0952
LRP	0.0148	0.0287	0.0947	0.1770	0.0516	0.0564	0.0267	0.0502	0.0758	0.0952	0.4259

Next we developed an algorithm that would generate random portfolio weight vectors *x*. It is important to make sure that this algorithm generates weight vectors *x* that cover all possible combinations of wealth allocation between 11 projects.

We believe this point needs some more explanation. In this context weight of any project in the portfolio is the fraction of this project's cost in the total wealth invested in the whole portfolio. It is not investor's percentage of participation in this project. Thus weight of project *i* (W_i) in portfolio is calculated according to the following equation:

$$\frac{f_i IN_i}{TW} = W_i \qquad\qquad (3.14)$$

where f_i is investor's percentage of participation in project *i*; IN_i, total cost of project *i*; *TW*, total cost of the portfolio.

Thus **Eq.3.14** sets constraints to the maximum weight of project *i* in the given portfolio that corresponds to f_i = 100% participation, and minimum weight that corresponds to f_i = minimum participation percentage (usually 1/16 or 1/12). That is algorithm that would generate weight vectors should generate them in discreet way, taking into account maximum and minimum weight values for each project.

In our implementation, for the reasons of simplicity and not knowing total amount that we are willing to invest in our portfolio, we generated continuous set of weight vectors with the only constraint for the weights to sum up to 1.0. The algorithm was implemented in MATLAB® computational environment. We ran this portfolio weights generation algorithm 10,000 times, and for each generated portfolio calculated its return and risk using **Eq. 3.12 and 3.13**, and plotted it as a dot on risk – return plane. The result is shown on the **Fig.3.20**.
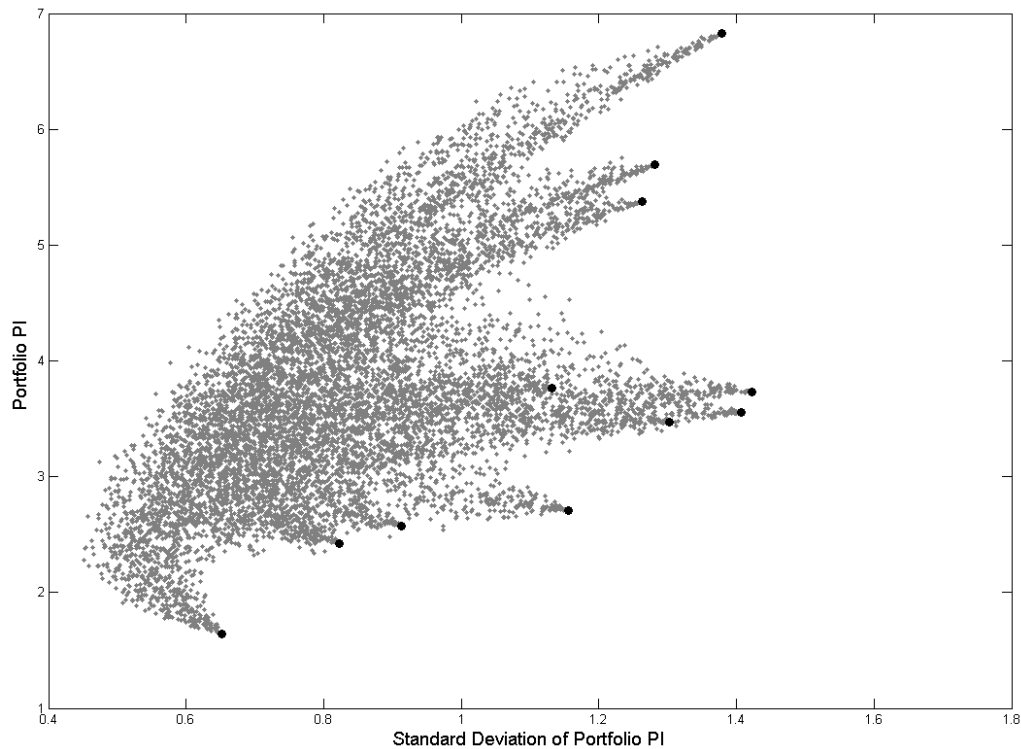
Fig. 3.21 - Opportunity set available to the investor (return measure – PI)

Fig.3.21 depicts the opportunity set available to the investors that has 11 projects in their portfolio. Positions of individual projects are depicted as black circles, while grey dots depict portfolios that consist of these projects. By means of changing wealth allocation from one asset to another, investors may attain any point within this set. However, of course, knowledgeable investors are interested only in efficient portfolios that provide the highest return for given level of risk or lowest risk for given level of return. Such portfolios form so-called "efficient frontier" and on this picture lie along the upper-left edge of opportunity set. Applying the same technique to other measures of return (GRR, NPV) we obtained analogous opportunity sets and efficient frontiers

shown in the **Figs. 3.22 – 3.23**. Thus we have completed development part of our model, and now we may proceed to the analysis of obtained results.
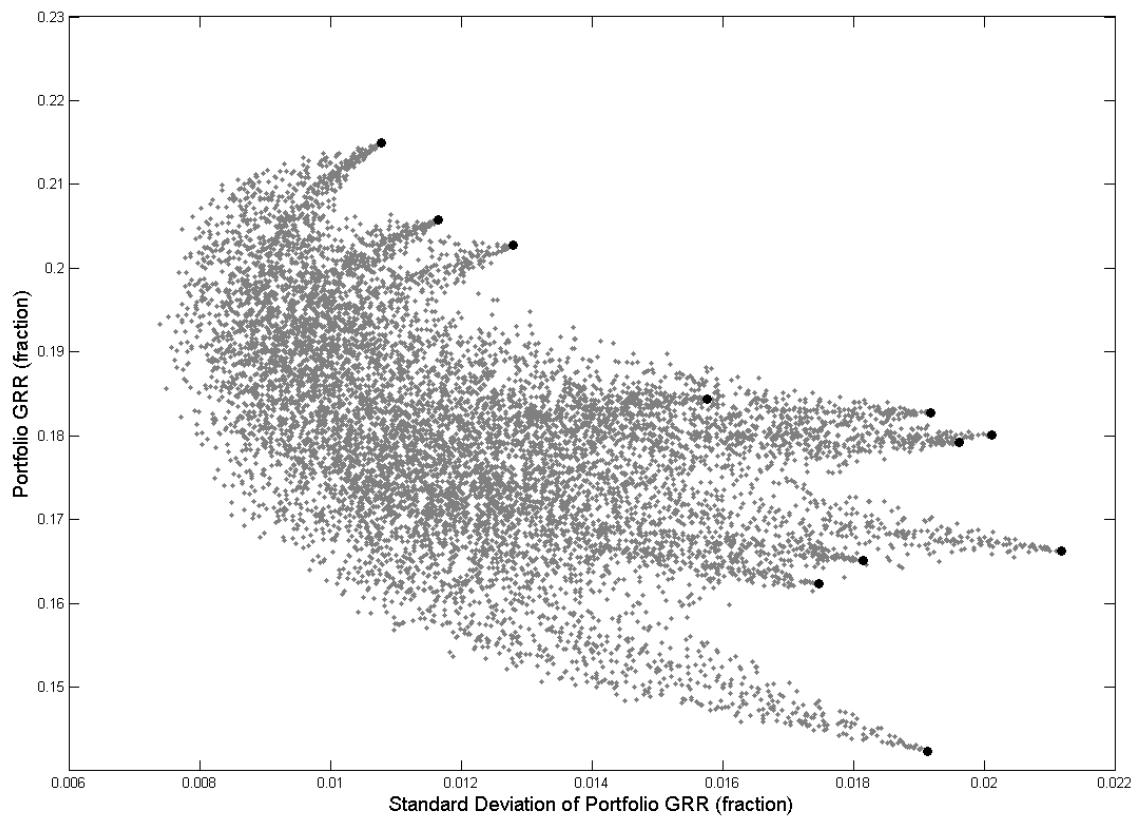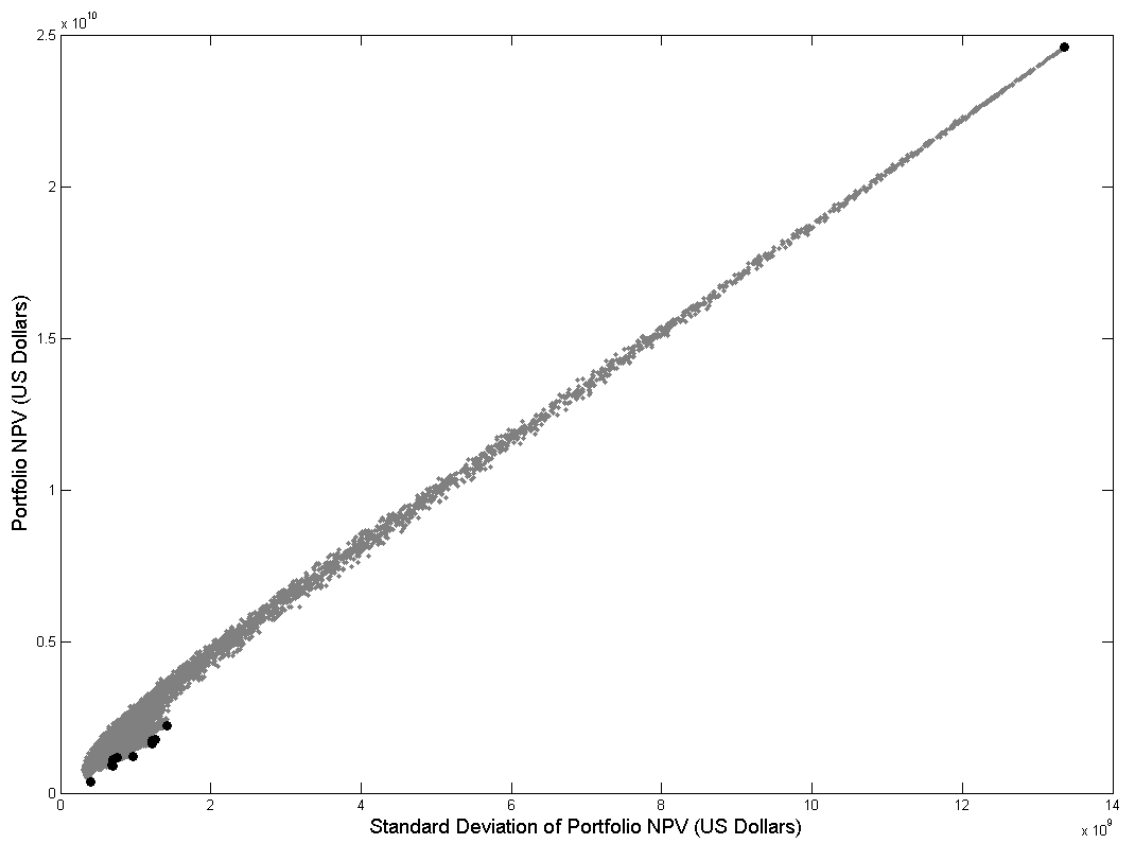
Fig. 3.22 - Opportunity set available to the investor (return measure – GRR)

Fig. 3.23 - Opportunity set available to the investor (return measure – NPV)

CHAPTER IV

RESULTS AND ANALYSIS

4.1 Analysis of Efficient Portfolios

A closer look at the efficient frontier is required. **Fig.4.1** shows efficient frontier when
return is measured by profitability index. This frontier consists of 95 efficient portfolios.
On **Fig.4.1** efficient frontier is divided on 5 regions based on portfolio risk. The first
region is least risk and fifth is the highest risk region.
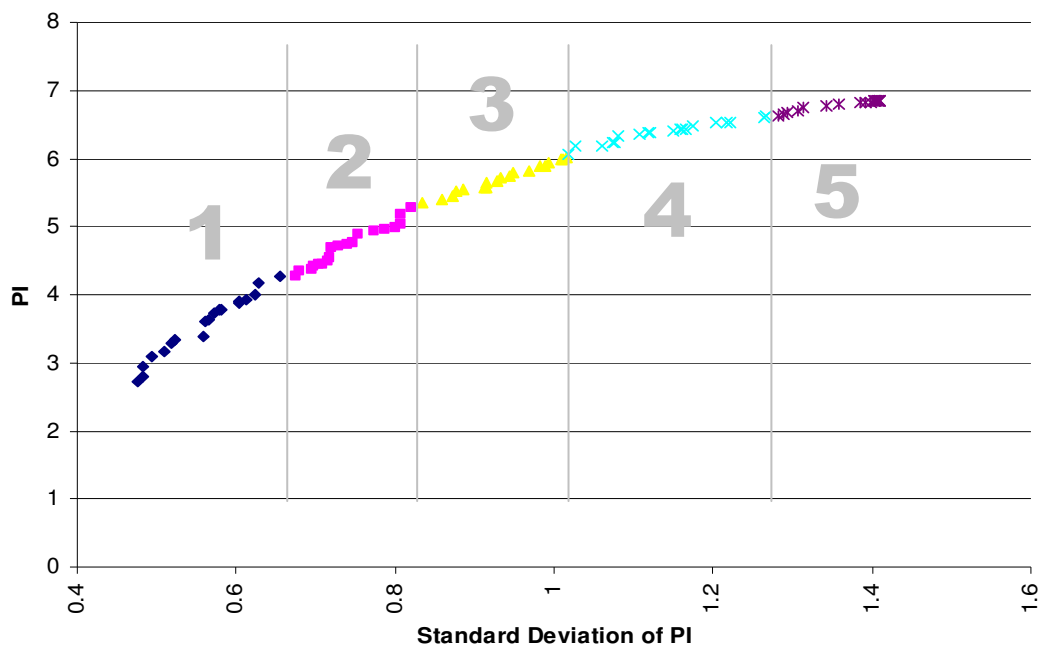
Fig. 4.1 – Efficient frontier (return measure – PI)

Fig.4.2 (a) – (e) show the portfolios' project structure in regions 1 – 5. Each line
corresponds to one portfolio, line shows fraction of total wealth invested in each project
in every particular portfolio. **Fig.4.2 (a)** corresponds to 10 portfolios in region 1 of

Fig.4.1, (b) corresponds to region 2, **(c)** to region 3, **(d)** to region 4, and **(e)** to region 5. Looking at the **Fig.4.2,** one can easily notice change in portfolio structure with the change in its risk level. Wealth in portfolios of lowest risk region 1 (**Fig.4.2 (a)**) is spread between 11 projects almost evenly. Though one can also notice that projects DLD and DLD_SOL on average have more weight than others. In region 2 (**Fig.4.2 (b)**) we can clearly see three "favorites", projects DLD, DLD_SOL, and HRP, with the former two weighting on average more than latter one. Portfolios in region 3, medium risk level, consist half of DLD and DLD_SOL, and half of HRP (**Fig.4.2 (c)**). In region 4 HRP dominates over all other projects, though DLD and DLD_SOL still have some weight (**Fig.4.2 (d)**). And in region 5, highest risk region, efficient portfolios almost totally consist of project HRP (**Fig.4.2 (e)**).

The same way we can analyze efficient frontier obtained using GRR as the measure of return. **Fig.4.3** shows this frontier divided on three regions, based on risk, and **Fig.4.4 (a) - (c)** show structures of efficient portfolios in three regions. Here again, efficient portfolios mainly consist of three projects DLD, DLD_SOL, and HRP. This consistency in result between two different measures of return can be explained if we'll take a look at the **Table 3.6**. These three projects (DLD, DLD_SOL, and HRP) have the highest expected values of PI and GRR as well as highest variance of these indicators. Therefore it is logical that they constitute a major part of investment in medium and high risk efficient portfolios. In low risk efficient portfolios, diversification is the major source of risk reduction, which is achieved by spreading wealth between all projects.
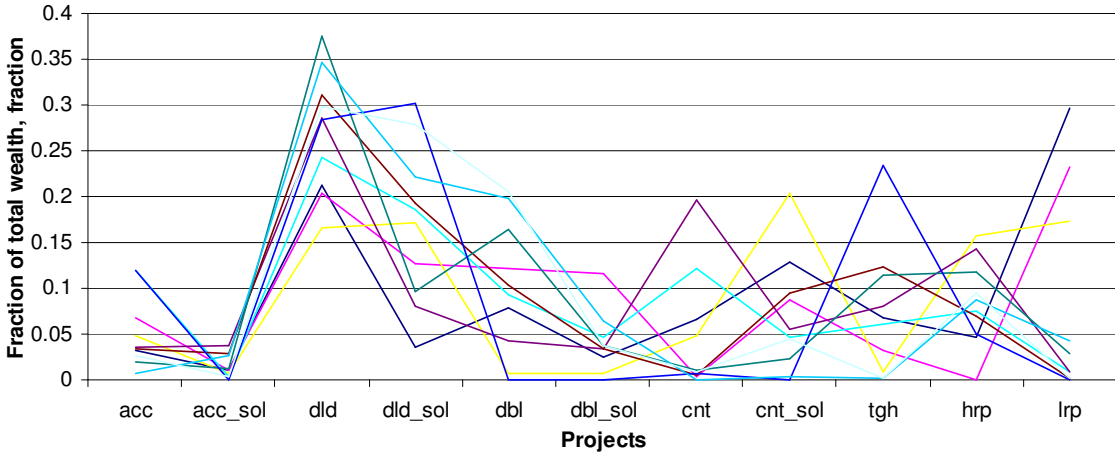
(a)
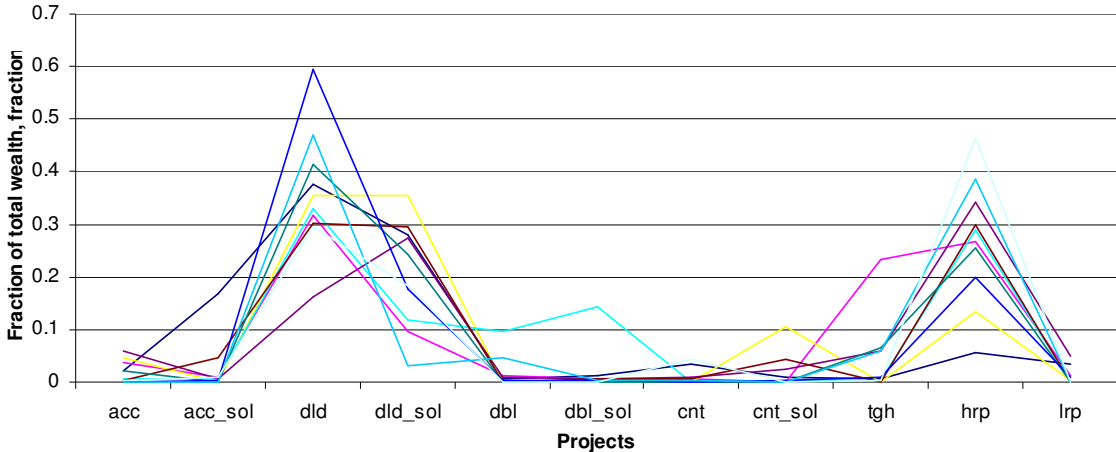
(b)
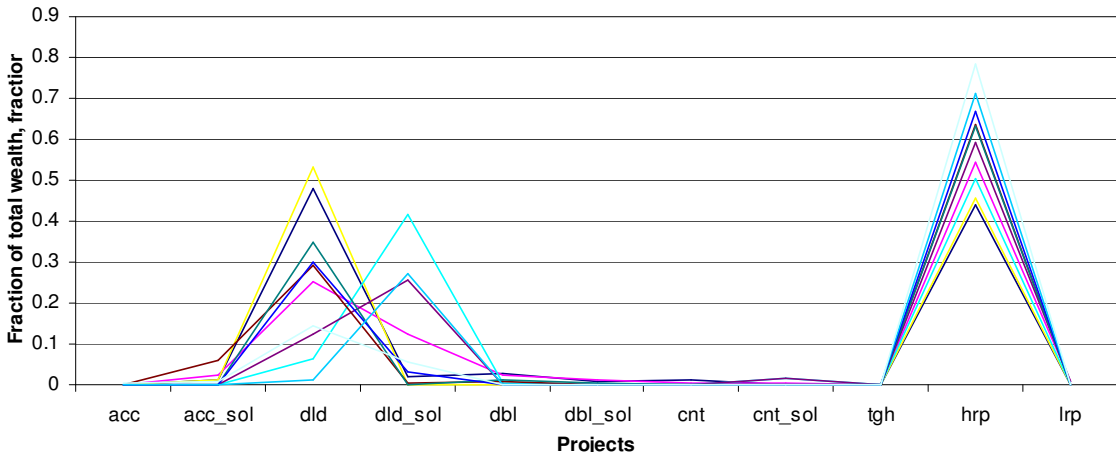
(c)

Fig. 4.2 - Efficient portfolio structures for region 1 (a), 2 (b), 3 (c), 4 (d), and 5 (e)

(d)

(e)

Fig. 4.2 - Continued

Fig. 4.3 – Efficient frontier (return measure – GRR)

(a)
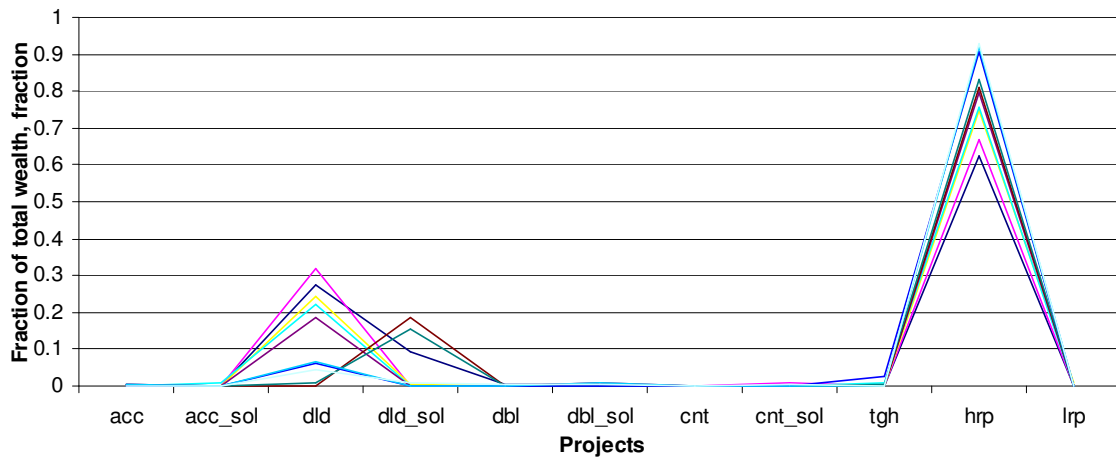
(b)
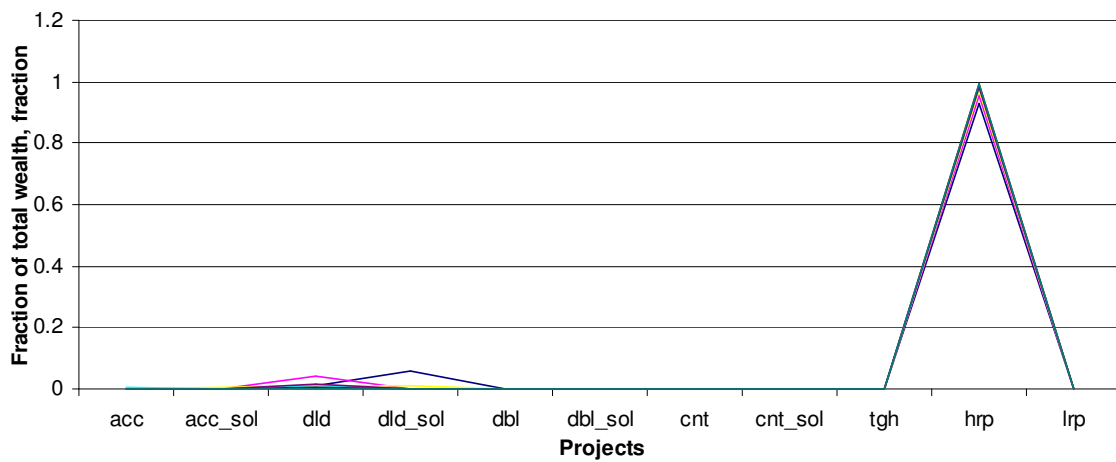
(c)

Fig. 4.4 - Efficient portfolio structures for region 1 (a), 2 (b), and 3 (c)

4.2 Sensitivity Analysis of Individual Projects

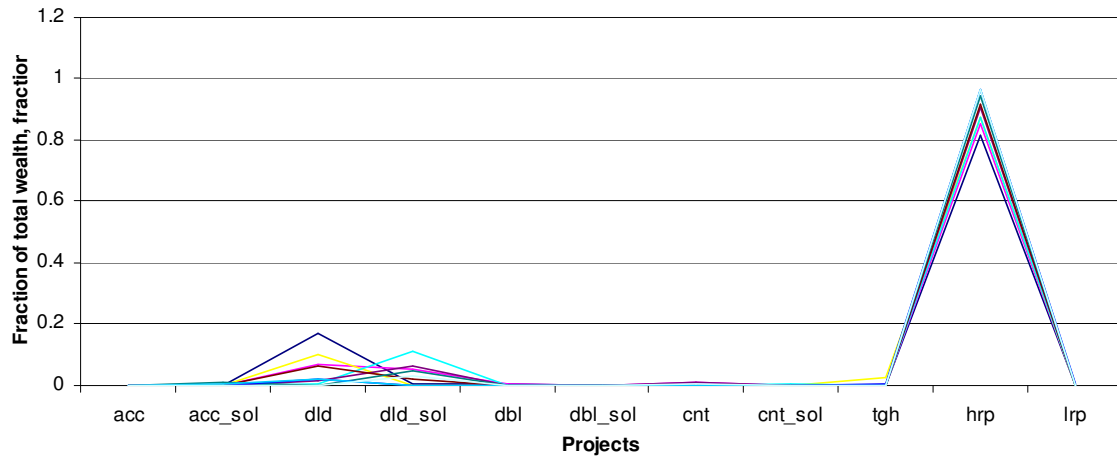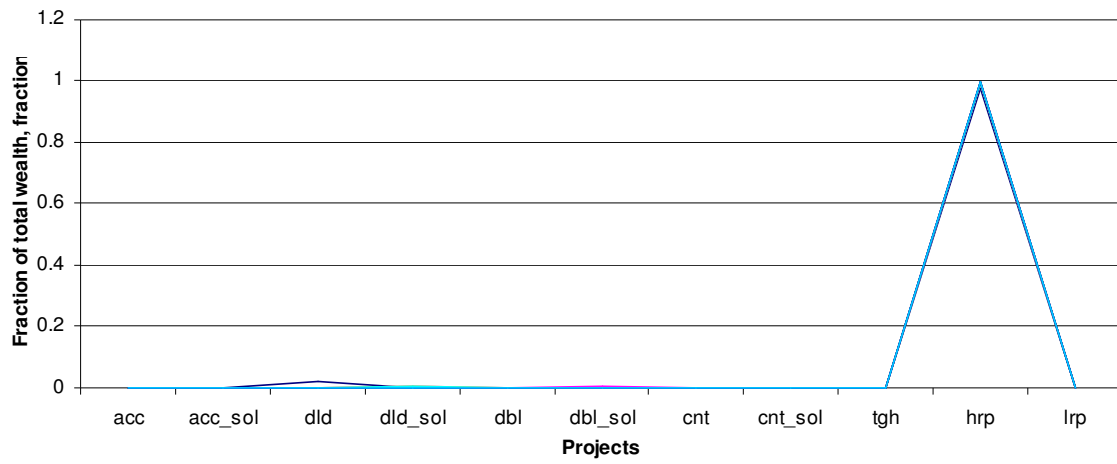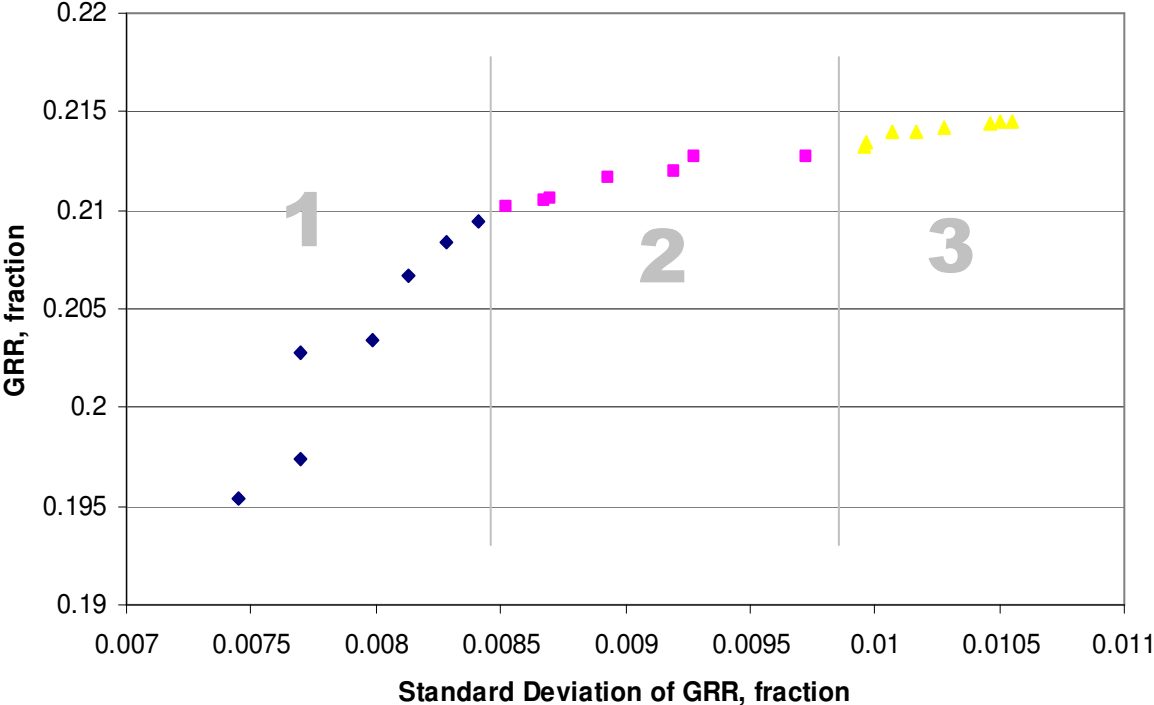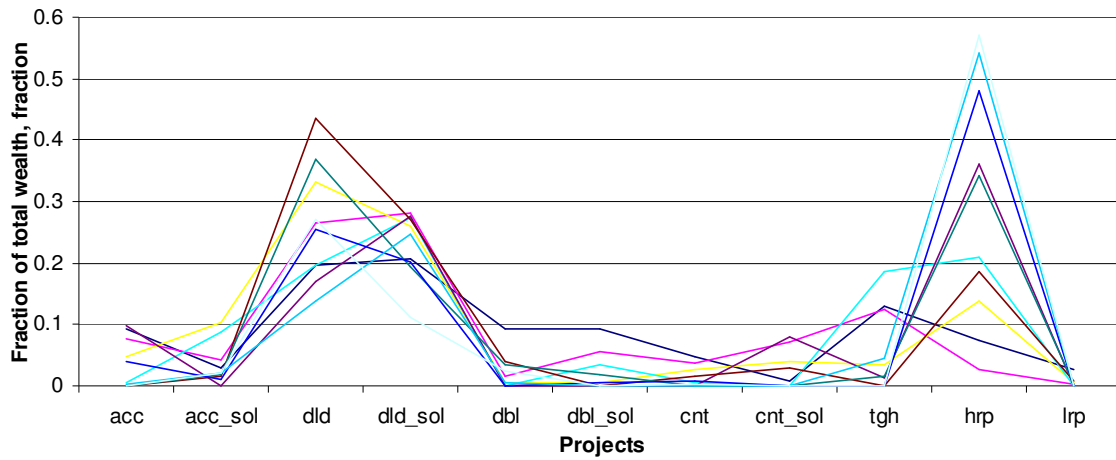Along with the analysis of efficient portfolios, we have conducted sensitivity analysis of individual projects. The objective of this analysis was to find an accurate estimation of which variables listed in **Table 3.4** is crucial for proper evaluation of project's performance.

"Spider-charts" were constructed, indicating the percentage change of input variable in consideration on horizontal axes and the percentage change in output variable (in our case NPV) on vertical axes. These charts are shown on **Fig.4.5 (a) – (f)**.

Fig.4.5 (a) and **(b)** show change in expected NPV as function of change in expected value of physical variables that are included in **Eq.3.5**: area and porosity. We can see that there is almost 1 to 1 positive relationship. Thus these variables are crucial, and their estimation should be done with corresponding level of consideration and regard.

Fig.4.5 (c) shows change in expected NPV as function of change in decline rate a in **Eq.3.7**. Surprisingly we found that this variable is not quite important: almost 100% change in decline rate led only to 6% change in NPV. This fact can be explained by the production profile type for which sensitivity analysis has been performed. We did it for accelerated profile type **Fig.3.12**. Because of its relatively long plateau period, decline period, the time when decline rate a really comes into play, is not very long, and as a result not very important. However for other types of production profiles, especially with short peaks followed by long decline periods, this variable should be estimated with attention.

Next comes analysis of cash flow variables: **Fig.4.5 (d)**, **(e)**, and **(f)** show sensitivity charts for amount of fixed CAPEX, amount of variable OPEX, and tax rate respectively. Among these three variables, tax rate has been found to be the most important one while CAPEX and OPEX change did not affect NPV so drastically. This can be explained by the fact that CAPEX and OPEX affect the project's performance only at early stages, leaving excess subsequent profits (if there are any) almost unaffected, while tax rate influences all profits evenly. Therefore, when the project's life is much longer than its payback period, project's eventual performance is more dependent on tax rate.
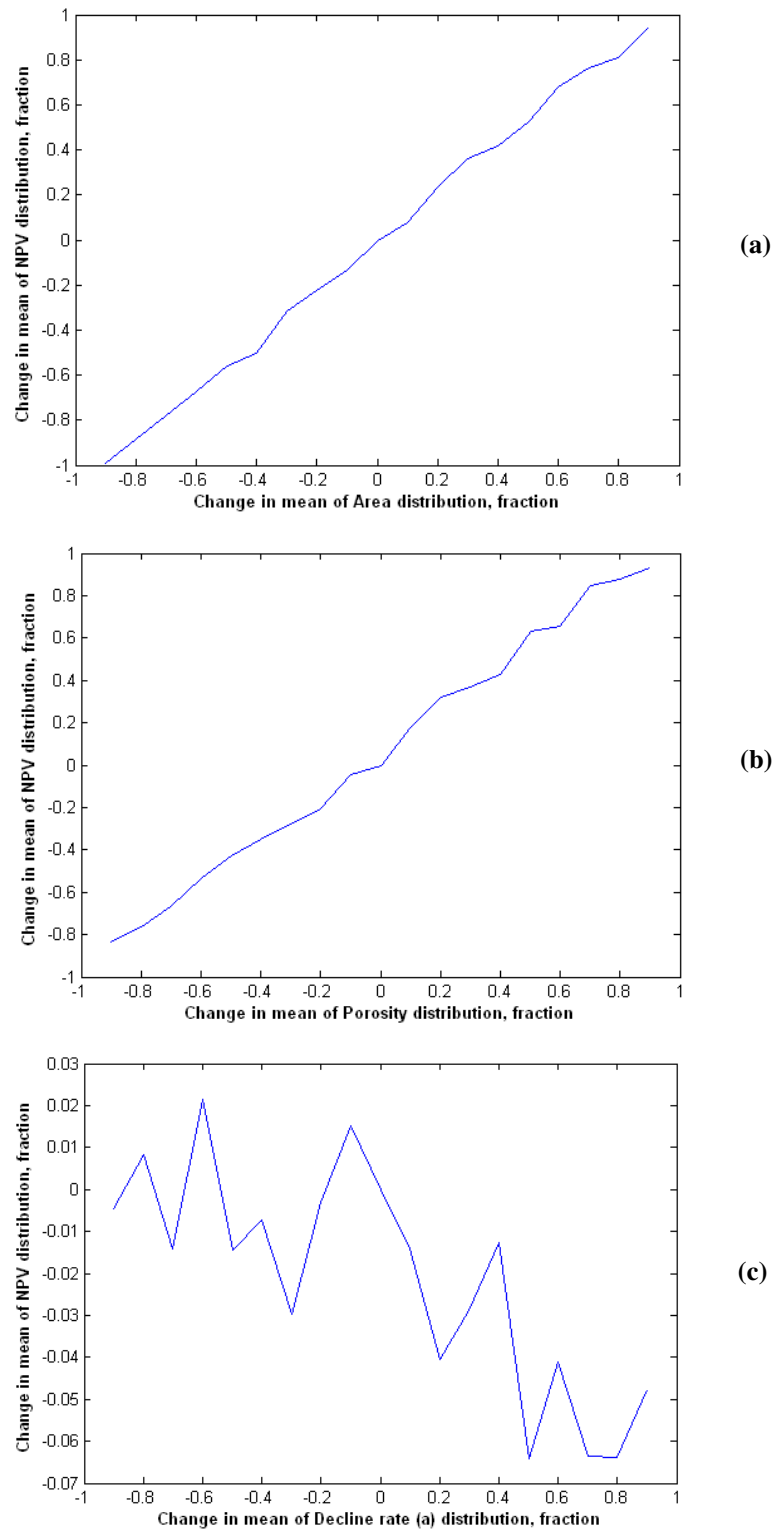
Fig. 4.5 - Project sensitivity analysis charts

(d)

(e)

(f)
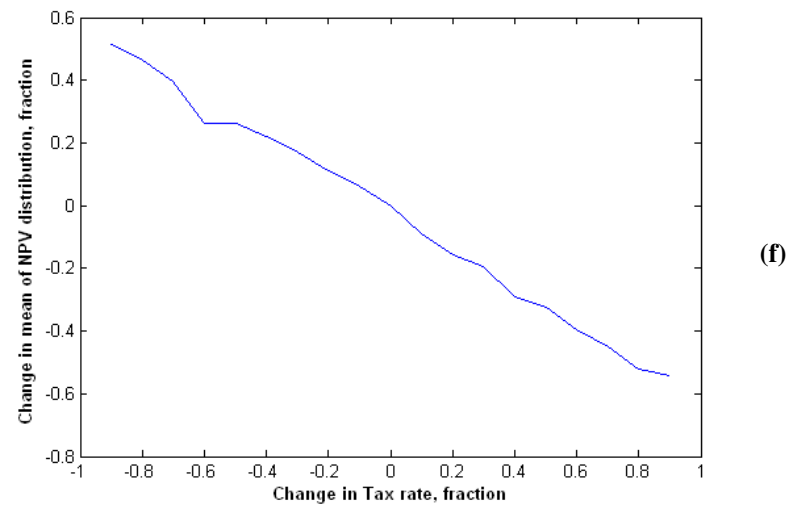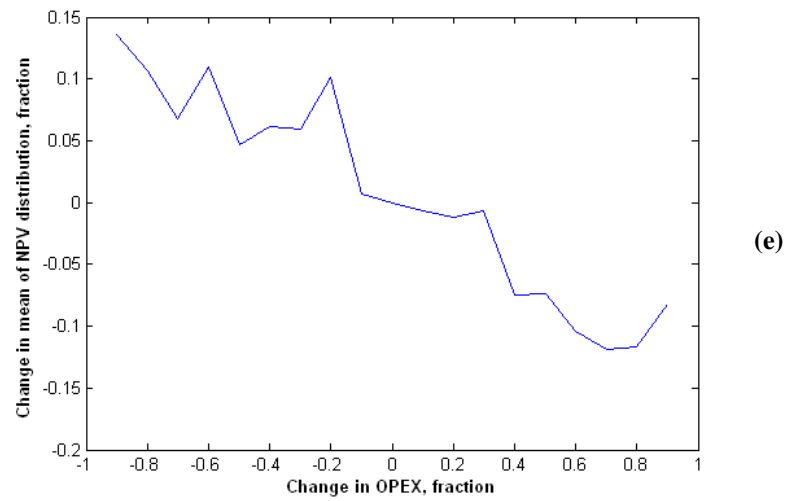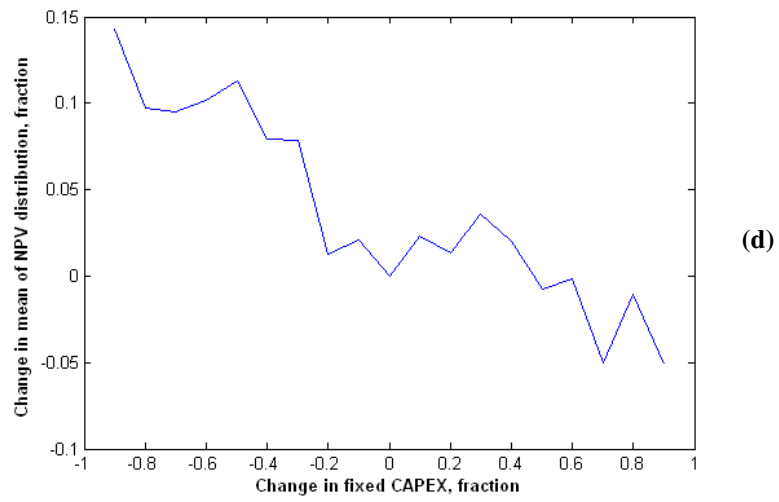
Fig. 4.5 - Continued

CHAPTER V

CONCLUSIONS

The objective of this research was development of an advanced computer based model for optimization of portfolio of petroleum producing assets. We wanted to develop this model applying best price estimation, production forecasting, and portfolio selection practices available now in the industry. The following are conclusions drawn as the result of this study.

1. For the first time in published literature, oil price forecasts generated using Sequential Gaussian Simulation (SGS) technique have been used to solve the portfolio optimization problem. As a result of this study, it has been found that SGS price forecasts can be and recommended to be successfully applied to portfolio optimization.

2. An advanced computer model for optimization of portfolio of petroleum assets has been developed. The model is based on Markowitz's Portfolio Theory and implemented in a MATLAB® computational environment.

3. A developed computer model allows optimization of portfolio using three different return measures (NPV, GRR, PI). The model has been successfully applied to the set of synthesized projects yielding reasonable solutions in all three return planes.

4. Analysis of obtained solutions has shown that the given computer model is robust and flexible in terms of input data and output results. Its modular

architecture allows further inclusion of complementary "blocks" that may solve optimization problem utilizing different (than considered) measures of risk and return as well as different input data formats.

5. Industry participants will benefit from using this proposed model by means of optimizing their portfolio of assets and expecting highest return for assumed level of risk.

REFERENCES

Allan, P.D. 2003. Consistent Performance: Reducing the Impacts of Price Uncertainty through Portfolio Management Practices. SPE-82001-PA. DOI: 10.2118/82001-PA.

Arps, J.J. 1944. Analysis of Decline Curves. SPE-945228-PA. DOI: 10.2118/945228-PA.

Ball, B.C., and Savage, S.L. 1999. Notes on Exploration and Production Portfolio Optimization. Available for download from http://www.stanford.edu/~savage.

Campbell, J.M. Jr., Campbell, J.M.Sr., and Campbell, R.A. 2001. *Analyzing and Managing Risky Investments*. John M. Campbell, Norman, Oklahoma.

Cronquist, C. 2001. *Estimation and Classification of Reserves of Crude Oil, Natural Gas, and Condensate*. Society of Petroleum Engineers, Richardson, Texas.

Davidson, L.B., and Davis, J.E. 1995. Simple, Effective Models for Evaluating Portfolios of Exploration and Production Projects. SPE-30669-PA. DOI: 10.2118/30669-PA.

Dias, F.S. 2003. Quadratic Programming Applied to Modern Portfolio Selection. MS Thesis. Universidade de Sao Paulo, Sao Paulo, Brazil.

Downey, M. December 1997. Business Side of Geology. *AAPG Explorer* **18** (4): 45-53.

Edwards, R.A., and Hewett, T.A. 1993. Applying Financial Portfolio Theory to the Analysis of Producing Properties. SPE-26392-PA. DOI: 10.2118/26392-PA.

Energy Information Administration. July, 2007. U.S. Crude Oil Daily Spot Prices. Energy Information Administration, Department of Energy. http://tonto.eia.doe.gov/dnav/pet/hist/rwtcd.htm.

Faulder, D.D., and Moseley, F.L. 2003. A "Top-Down" Approach for Applying Modern Portfolio Theory to Oil and Gas Property Investments. SPE-82028-PA. DOI: 10.2118/82028-PA.

Fetkovich, M.J., Fetkovich, E.J., and Fetkovich, M.D. 1994. Useful Concepts for Decline-Curve Forecasting, Reserve Estimation and Analysis. SPE-28628-PA. DOI: 10.2118/28628-PA.

Gallun, R.A., Wright, C.J., Nichols, L.M., and Stevenson, J.W. 2001. *Fundamentals of Oil & Gas Accounting, 4th edition*. PennWell, Tulsa, Oklahoma.

Hertz, D.B. 1968. Investment Policies That Pay Off. *Harvard Business Review* **46** (1): 96-108.

Hightower, M.L., and David, A. 1991. Portfolio Modeling: A Technique for Sophisticated Oil and Gas Investors. SPE-22016-PA. DOI: 10.2118/22016-PA.

Holmes, J.C. *et al*. 2006. Quantification of Oil Price Uncertainty in Economic Evaluation Using Sequential Gaussian Simulation. *Petroleum Science and Technology* **24**: 943-962.

Houlding, S.W. 2000. *Practical Geostatistics: Modeling and Spatial Analysis*. Springer-Verlag, Berlin, Germany.

Kanevski, M., and Maignan, M. 2004. *Analysis and Modeling of Spatial Environmental Data*. EPFL Press, Basel, Switzerland.

King, S.L. 2000. Sequential Gaussian Simulation vs. Simulated Annealing for Locating Pockets of High-value Commercial Trees in Pennsylvania. *Annals of Operations Research* **95**: 177-203.

Markowitz, H. 1952. Portfolio Selection. *The Journal of Finance* **7** (1): 77-91.

McVean, J.R. 2000. The Significance of Risk Definition on Portfolio Selection. SPE-62966-PA. DOI: 10.2118/62966-PA.

Mian, M.A. 2002. *Project Economics and Decision Analysis, Volume 1: Deterministic Models*. PennWell, Tulsa, Oklahoma.

Montgomery, D.C., and Runger, G.C. 2007. *Applied Statistics and Probability for Engineers, 4th edition*. John Wiley & Sons, Hoboken, New Jersey.

Olsen, G.T. *et al.* 2005. Price Uncertainty Quantification Models Advance Project Economic Evaluations. SPE-94610-PA. DOI: 10.2118/94610-PA.

Seba, R.D. 1998. *Economics of Worldwide Petroleum Production*. Oil and Gas Consultants International, Tulsa, Oklahoma.

Sharpe, W.F. 1964. Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *The Journal of Finance* **19** (3): 425-442.

Slocum, T. 2001. No Competition: Industry Mergers Provide Higher Profits, Leave Consumer with fewer Choices. Available for download from http://www.citizen.org/documents/nocompetition.PDF.

U.S. Department of Labor. July, 2007. Consumer Price Index for All Urban Consumers: All Items. U.S. Department of Labor: Bureau of Labor Statistics, http://stats.bls.gov/cpi/home.htm#data.

APPENDIX A

MATLAB CODE FOR VARIOGRAM GENERATION

AND SGS PRICE FORECASTING

```
price_data=[1.094331707;-0.256429712;-1.112616861;-1.024592393;-
0.215515948;-0.771389342;-1.396053408;-0.350197509;-0.436190648;-
0.458176841;-0.329121979;-0.114640397;0.607301574;0.360793305;
0.514156101;0.536996776;0.706047289;0.784853134;0.913249927;0.798460728
;0.655874678;0.718868053;0.491580583;0.004973489;-0.054735476;-
0.144734328;-0.287395477;0.124658832;-0.034821317;-0.277044376;-
0.548522283;-0.56012112;-0.854447399;-1.131281786;-1.04155283;-
0.41441333;-0.004973489;-0.074671366;0.371429764;0.693341543;
0.469252884;0.425276777;0.350197509;-0.01492096;0.246163647;
0.403598571;0.287395477;0.571795808;0.82613034;0.731808084;0.329121979;
-0.164868332;-0.23592346;-0.607301574;-0.185069399;1.076405303;
1.297542929;1.345166634;1.189794977;1.058818296;0.928449208;0.084649956
;-0.084649956;0.164868332;0.23592346;-0.024869907;0.256429712;
0.318639364;0.392830814;0.631401894;0.480386797;-0.225707954;-
0.403598571;-0.371429764;-0.392830814;-0.134689794;0.054735476;
0.382108412;0.215515948;0.074671366;0.195197808;0.144734328;-
0.154793486;-0.382108412;-0.480386797;-0.266722875;-0.195197808;-
0.246163647;-0.318639364;-0.502836307;-0.718868053;-0.668259041;-
0.928449208;-0.643590103;-1.169844649;-1.575767629;-1.479941389;-
1.510356896;-1.542237572;-1.252692984;-0.868865622;-0.61930677;-
0.491580583;-0.731808084;-1.058818296;-0.991526475;-0.913249927;-
1.15034938;-0.943866065;-0.758063952;-0.812217801;-0.525542166;-
0.595383277;-0.883466774;-1.210230452;-1.007921679;-0.975393555;-
1.189794977;-1.076405303;-0.706047289;-0.82613034;-0.74487183;-
0.360793305;0.064700206;-0.447156725;-0.571795808;-0.425276777;-
0.297777464;0.094636982;0.277044376;0.01492096;0.308191647;0.297777464;
-0.308191647;-0.536996776;-0.798460728;-0.583548958;-0.95951059;-
0.840204652;-0.693341543;-0.784853134;-0.514156101;-0.655874678;-
1.23118423;-1.450836487;-1.611169162;-1.688737002;-1.64871347;-
```

```
1.778042802;-2.01742871;-1.946561721;-2.100165493;-1.731664396;-
1.884304423;-2.200410581;-2.880674359;-2.329336053;-2.514954878;-
1.828594899;-1.42291118;-1.37016654;-1.345166634;-0.898258749;-
0.631401894;-0.17495994;-0.469252884;-0.044776177;0.185069399;
0.436190648;0.74487183;0.771389342;0.034821317;0.583548958;0.868865622;
0.680746755;0.812217801;0.95951059;0.943866065;1.024592393;0.458176841;
0.643590103;0.61930677;0.17495994;0.225707954;0.41441333;0.205346281;-
0.064700206;0.154793486;-0.094636982;-0.680746755;-1.297542929;-
1.320980289;-1.274797425;-1.094331707;-0.339640886;-
0.104633456;0.044776177;-0.205346281;0.024869907;0.266722875;
0.525542166;0.339640886;-0.124658832;0.447156725;0.840204652;
0.991526475;0.854447399;0.104633456;0.114640397;0.548522283;0.56012112;
0.668259041;0.134689794;0.502836307;0.595383277;0.758063952;0.883466774
;0.898258749;1.007921679;0.975393555;1.112616861;1.04155283;1.131281786
;1.169844649;1.210230452;1.396053408;1.274797425;1.15034938;1.23118423;
1.252692984;1.42291118;1.37016654;1.320980289;1.450836487;1.611169162;1
.884304423;2.01742871;1.778042802;1.542237572;1.575767629;1.946561721;1
.688737002;1.828594899;2.100165493;2.329336053;2.200410581;2.880674359;
2.514954878;1.731664396;1.479941389;1.510356896;1.64871347];


variogramm_data=0;
for step=1:250
    n=252-step;
    total_diff_sq=0;
    head=0;
    tail=0;
    for j=1:n
        stepped_out=j+step;
        total_diff_sq=total_diff_sq+(price_data(stepped_out,1)-
price_data(j,1))^2;
        head(j,1)=price_data(stepped_out,1);
        tail(j,1)=price_data(j,1);
    end
    variogramm_data(step,1)=step;
    variogramm_data(step,2)=total_diff_sq/(2*n);
```

```
variogramm_data(step,3)=(total_diff_sq/(2*n))/(std(head)*std(tail));
end
variogramm_data
plot(variogramm_data(:,1),variogramm_data(:,2),'+');


xlswrite('variogramm_data', variogramm_data);

prices_cdf=[13.96603659 0.001984127;14.75117468 0.005952381;15.40279439
0.009920635;15.99634146 0.013888889;16.65591912 0.017857143;17.00673219
0.021825397;17.5429816  0.025793651;17.83635697 0.029761905;18.00865653
0.033730159;18.51592615 0.037698413;18.56214198 0.041666667;18.84630019
0.045634921;19.09759556 0.049603175;20.05512376 0.053571429;20.08311385
0.057539683;20.19375596 0.061507937;20.40084815 0.06547619;20.80283951
0.069444444;20.91814011 0.073412698;21.17065455 0.077380952;21.3410411
0.081349206;21.51561974 0.08531746;21.75845969 0.089285714;22.05694476
0.093253968;22.30361283 0.097222222;22.50977929 0.101190476;22.51057065
0.10515873;22.90398762 0.109126984;22.93241967 0.113095238;22.95936031
0.117063492;23.00547701 0.121031746;23.13218437 0.125;23.19520868
0.128968254;23.28177493 0.132936508;23.60980237 0.136904762;23.61992193
0.140873016;23.63362416 0.14484127;23.73920133 0.148809524;23.81536765
0.152777778;23.84548197 0.156746032;23.93504685 0.160714286;24.06026161
0.16468254;24.27650219 0.168650794;24.31845023 0.172619048;24.38881215
0.176587302;24.39147826 0.180555556;24.40541516 0.18452381;24.46261498
0.188492063;24.49255088 0.192460317;24.65690756 0.196428571;24.74976918
0.200396825;24.78130293 0.204365079;24.79371769 0.208333333;24.846625
0.212301587;24.84850746 0.216269841;24.88681359 0.220238095;24.93297405
0.224206349;24.95053109 0.228174603;24.99382749 0.232142857;25.00139889
0.236111111;25.00165365 0.240079365;25.0835514  0.244047619;25.09201346
0.248015873;25.16909972 0.251984127;25.21251238 0.255952381;25.24238456
0.259920635;25.76067187 0.263888889;26.07666441 0.267857143;26.08405573
0.271825397;26.22470046 0.275793651;26.22644195 0.279761905;26.31389527
0.283730159;26.42983966 0.287698413;26.50762712 0.291666667;26.51469925
0.295634921;26.52457067 0.299603175;26.70219603 0.303571429;26.71540915
0.307539683;26.79304054 0.311507937;27.06310078 0.31547619;27.27124479
0.319444444;27.28511797 0.323412698;27.33273193 0.327380952;27.35429353
```

0.331349206;27.43037652 0.33531746;27.47700748 0.339285714;27.4968963
0.343253968;27.54730159 0.347222222;27.58407042 0.351190476;27.77855177
0.35515873;27.78821175 0.359126984;27.82812785 0.363095238;27.84113611
0.367063492;27.84583862 0.371031746;27.95770833 0.375;28.13277184
0.378968254;28.14917197 0.382936508;28.18241379 0.386904762;28.38939574
0.390873016;28.43030856 0.39484127;28.44717153 0.398809524;28.4571727
0.402777778;28.49309542 0.406746032;28.55660377 0.410714286;28.57904236
0.41468254;28.64258065 0.418650794;28.65531796 0.422619048;28.66212471
0.426587302;28.74230999 0.430555556;28.89801088 0.43452381;28.94648801
0.438492063;29.3019879 0.442460317;29.30663317 0.446428571;29.32945394
0.450396825;29.44744565 0.454365079;29.54767338 0.458333333;29.7868169
0.462301587;29.79094955 0.466269841;29.89506193 0.470238095;29.9525618
0.474206349;29.95523397 0.478174603;29.9940547 0.482142857;30.02011956
0.486111111;30.04676991 0.490079365;30.14022508 0.494047619;30.17789212
0.498015873;30.20005199 0.501984127;30.22538219 0.505952381;30.25317399
0.509920635;30.31714953 0.513888889;30.34856507 0.517857143;30.34956272
0.521825397;30.45793192 0.525793651;30.65061922 0.529761905;30.70478455
0.533730159;30.75108709 0.537698413;30.86159609 0.541666667;30.86288357
0.545634921;30.9368927 0.549603175;30.94776815 0.553571429;30.97694268
0.557539683;31.11699155 0.561507937;31.13699259 0.56547619;31.26866894
0.569444444;31.29518717 0.573412698;31.33700497 0.577380952;31.34316263
0.581349206;31.34952924 0.58531746;31.48400681 0.589285714;31.68797337
0.593253968;31.7276244 0.597222222;31.75382353 0.601190476;31.81067185
0.60515873;31.8173891 0.609126984;31.90882166 0.613095238;31.97499369
0.617063492;32.10223203 0.621031746;32.13687225 0.625;32.14610938
0.628968254;32.15420994 0.632936508;32.16441579 0.636904762;32.21178058
0.640873016;32.32782895 0.64484127;32.32844667 0.648809524;32.3382284
0.652777778;32.44944 0.656746032;32.65988694 0.660714286;32.68247173
0.66468254;32.68608437 0.668650794;32.79110866 0.672619048;32.96491671
0.676587302;32.98307067 0.680555556;32.98710335 0.68452381;32.99158716
0.688492063;33.05946004 0.692460317;33.09086022 0.696428571;33.12334256
0.700396825;33.62733274 0.704365079;33.71764578 0.708333333;33.77980403
0.712301587;33.8203336 0.716269841;33.91606538 0.720238095;33.93512432
0.724206349;34.05945701 0.728174603;34.12505997 0.732142857;34.16774052
0.736111111;34.3175977 0.740079365;34.45064685 0.744047619;34.642882
0.748015873;34.76484919 0.751984127;34.76636141 0.755952381;34.80915705

```
0.759920635;34.84998261 0.763888889;35.02196232 0.767857143;35.1117654
0.771825397;35.14273171 0.775793651;35.46355713 0.779761905;35.81013263
0.783730159;36.01544815 0.787698413;36.50618056 0.791666667;36.58325139
0.795634921;36.75682698 0.799603175;36.93237575 0.803571429;37.44184257
0.807539683;37.56786761 0.811507937;37.78846652 0.81547619;37.94195595
0.819444444;38.05242152 0.823412698;38.46630973 0.827380952;39.56587963
0.831349206;39.57390608 0.83531746;39.79358283 0.839285714;39.81810956
0.843253968;39.91928736 0.847222222;40.58410365 0.851190476;41.14427504
0.85515873;42.2634816  0.859126984;42.33553522 0.863095238;43.2367234
0.867063492;43.38114918 0.871031746;45.58989529 0.875;47.83959873
0.878968254;48.87036704 0.882936508;48.92185752 0.886904762;49.67058329
0.890873016;50.95264814 0.89484127;51.23753798 0.898809524;51.38539514
0.902777778;51.67365879 0.906746032;54.80687265 0.910714286;55.30969477
0.91468254;56.61876777 0.918650794;57.01533889 0.922619048;58.49501029
0.926587302;58.57073435 0.930555556;59.08   0.93452381;59.08120482
0.938492063;60.67276316 0.942460317;61.21439589 0.946428571;62.05224814
0.950396825;62.71777105 0.954365079;63.14291319 0.958333333;63.19953722
0.962301587;63.66805234 0.966269841;67.11863869 0.970238095;67.15387195
0.974206349;67.39339104 0.978174603;70.1350951  0.982142857;70.70474074
0.986111111;70.94546898 0.990079365;72.42983784 0.994047619;74.00659438
0.998015873];


a = 26; Co=0; C=1;
prices=(-251:0)'; prices(:,2)=price_data;
m=252;
set=(1:m)';
set(:,2)=floor(rand(m,1)*1000);
sorted_set=sortrows(set,2);
route=sorted_set(:,1);

for k=1:m
n=length(prices(:,1));
Gamm_matrx=0;
for i=1:n
    for j=1:n
        if prices(j,1)==prices(i,1)
```

```
            Gamm_matrx(i,j)=Co;
        elseif abs(prices(j,1)-prices(i,1))>a
            Gamm_matrx(i,j)=1;
        else
            Gamm_matrx(i,j)=Co+C*(3*abs(prices(j,1)-prices(i,1))/(2*a)-
.5*(abs(prices(j,1)-prices(i,1))/a)^3);
        end
    end
end
Gamm_matrx(:,n+1)=1:1;
Gamm_matrx(n+1,:)=1:1;
Gamm_matrx(n+1,n+1)=0;


vectr=0;
for j=1:n
    if abs(prices(j,1)-route(k))>a
        vectr(j,1)=1;
    else
        vectr(j,1)=Co+C*(3*abs(prices(j,1)-route(k))/(2*a)-
.5*(abs(prices(j,1)-route(k))/a)^3);
    end
end


vectr(n+1,1)=1;
weights = Gamm_matrx\vectr;


expmean=0;
varns=0;
for j=1:n
    expmean=expmean+weights(j)*prices(j,2);
    varns=varns+weights(j)*vectr(j,1);
end
varns=varns+weights(n+1);
prices(n+1,1)=route(k); prices(n+1,2)=normrnd(expmean, varns);
end
```

```
prices=sortrows(prices,1);
prices(:,3)=normcdf(prices(:,2),0,1);
frcst(1,1)=0;
v=length(prices(:,1));
for i=253:v
    reach_end=0;
    row=0;
    for j=1:252
        if prices(i,3) < prices_cdf(j,2)
            row=j;
            break
        end
        if j==252
            reach_end=1;
        end
    end
    if row==1
        frcst((i-
252),1)=(prices(i,3))*(prices_cdf(row,1))/(prices_cdf(row,2))+0;
        frcst((i-252),2)=frcst((i-252),1)*(1.002667)^(i-252);
    elseif reach_end==1
        frcst((i-252),1)=(prices(i,3)-prices_cdf(252,2))*(100-
prices_cdf(252,1))/(1-prices_cdf(252,2))+prices_cdf(252,1);
        frcst((i-252),2)=frcst((i-252),1)*(1.002667)^(i-252);
    else
        frcst((i-252),1)=(prices(i,3)-prices_cdf((row-
1),2))*(prices_cdf(row,1)-prices_cdf((row-1),1))/(prices_cdf(row,2)-
prices_cdf((row-1),2))+prices_cdf((row-1),1);
        frcst((i-252),2)=frcst((i-252),1)*(1.002667)^(i-252);
    end
end
xlswrite('forecast',frcst);
```

MATLAB CODE FOR PRODUCTION AND

CASH FLOW FORECASTING FOR PROJECT "ACC"

```
m=1000;
n=m;
Sw = normrnd(.3,.09,n,1);
% making shure that Sw is within 0 -> 1
for i=1:n
    if Sw(i)<0
        Sw(i)=0;
    end
    if Sw(i)>1
        Sw(i)=.9999;
    end
end


Bo = unifrnd(1.05,1.5,n,1);
Area_h_mus = [0 0];
Area_h_sigmas = [1 0.6; 0.6 1];
Area_h = mvnrnd(Area_h_mus,Area_h_sigmas,n);
Area = Area_h(:,1)*300+3600;
h = Area_h(:,2)*0.165526349+4.08064508;
h = exp(h);


phi_Cr_mus = [0 0];
phi_Cr_sigmas = [1 0.6; 0.6 1];
phi_Cr = mvnrnd(phi_Cr_mus,phi_Cr_sigmas,n);
phi = phi_Cr(:,1)*.08+.3;
Cr = phi_Cr(:,2)*.28012669-1.0890584;
Cr = exp(Cr);
% making shure that Cr and phi are within 0 -> 1
for i=1:n
    if Cr(i)<0
        Cr(i)=0;
```

```matlab
        end
        if Cr(i)>1
            Cr(i)=.9999;
        end
        if phi(i)<0
            phi(i)=0;
        end
        if phi(i)>1
            phi(i)=.9999;
        end
    end
    for i=1:n
        N(i,1)= 7758 * Area(i,1) * h(i,1) * phi(i,1) * (1 - Sw(i,1)) * Cr
(i,1) / Bo(i,1);
        qi(i,1) = N(i,1)*.08/12;
        t_reach_plateau(i,1) = round(unifrnd(0,24,1,1));
        t_at_plateau(i,1) = round(unifrnd(84,108,1,1));
        t_delay(i,1) = round(unifrnd(0,1,1,1));
    end
    %hist(N,30);
    q1_N=prctile(N,30);
    q2_N=prctile(N,50);

    a_month = unifrnd(.10/12,.20/12,n,1);
    Production = 0;
    for i=1:n
        j=1;
        Total_Production=0;
        while j<=t_delay(i,1)
            Production(j,i)=0;
            Total_Production=Total_Production + Production(j,i);
            j=j+1;
        end
        while j-t_delay(i,1)<=t_reach_plateau(i,1)
            term_rand=0;%normrnd(0,3000,1,1);
```

```matlab
        Production(j,i)=(qi(i,1)/t_reach_plateau(i,1))*(j-
t_delay(i,1))+term_rand;
        Total_Production=Total_Production + Production(j,i);
        j=j+1;
    end
    while j-t_delay(i,1)-t_reach_plateau(i,1)<=t_at_plateau(i,1)
        term_rand=0;%normrnd(0,3000,1,1);
        Production(j,i)=qi(i,1)+term_rand;
        Total_Production=Total_Production + Production(j,i);
        j=j+1;
    end
    diff = N(i,1)-Total_Production;
    while diff > 30000
        term_rand=0;%normrnd(0,3000,1,1);
        Production(j,i) = qi(i,1) * exp(-a_month(i,1)*(j-t_delay(i,1)-
t_reach_plateau(i,1)-t_at_plateau(i,1)))+term_rand;
        Total_Production = Total_Production + Production(j,i);
        diff = N(i,1)-Total_Production;
    if j==252
        percent_produced=100*Total_Production/N(i,1);
        break
    end
    j=j+1;
    end

    if i==m
        break
    end
end

average_path=mean(Production');
Production(:,m+1) = average_path';
xlswrite('production_data_accelerated', Production);


% -------------------- CASH FLOW MODULE ------------------------------
prices_inflated = [% --- here comes 50 SGS inflated price forecasts% ];
```

```
Cash_flow = 0;
Cum_Cash_flow = 0;
seis_succ = 0;
drill_succ = 0;
investments=0;
seismic = -20000000;
exp_drilling = -100000000;
prod_drilling = -200000000;
opex = 4;
capex = 1;
tax_rate=.36;
price_path = round(unifrnd(.51,50.49,n,1));

for i=1:m
    capex_vctr=0;
    prod_cf_vctr =0;
    depreciation1_vctr=0;
    depreciation2_vctr=0;
    depreciation3_vctr=0;
    total_deprtn_vcrt=0;
    prod_cf_vctr=0;
    capex_vctr(1,1) = seismic;
    depreciation1_vctr(1,1) = 0; depreciation1_vctr(2:85,1) =
(seismic/7)/12; depreciation1_vctr(97,1)=0;
    if N(i)<q1_N
        seis_succ(i,1)=0;
        drill_succ(i,1)=0;
    else
        seis_succ(i,1)=1;
    end
    if seis_succ(i,1)==1
        capex_vctr(7,1) = exp_drilling;
        depreciation2_vctr(1:7,1) = 0; depreciation2_vctr(8:91,1) =
(exp_drilling/7)/12; depreciation2_vctr(97,1)=0;
        if N(i)<q2_N
```

```
          drill_succ(i,1)=0;
      else
          drill_succ(i,1)=1;
      end
      if drill_succ(i,1) ==1
          capex_vctr(13,1) = prod_drilling-N(i,1)*capex;
          depreciation3_vctr(1:13,1) = 0; depreciation3_vctr(14:97,1)
= (capex_vctr(13,1)/7)/12;
          for j=1:252
prod_cf_vctr(13+j,1)=Production(j,i)*prices_inflated(j,price_path(i,1))
-Production(j,i)*opex;
          end
      else
          capex_vctr(13,1) = 0;
          depreciation3_vctr(97,1) = 0;
      end
  else
      capex_vctr(13,1) = 0;
      depreciation2_vctr(97,1) = 0;
      depreciation3_vctr(97,1) = 0;
  end
  capex_vctr(300,1)=0;
total_deprtn_vcrt=depreciation1_vctr+depreciation2_vctr+depreciation3_v
ctr; total_deprtn_vcrt(300,1)=0;
  prod_cf_vctr(300,1)=0;


  for j=1:300
      Cash_flow(j,i)=capex_vctr(j,1)+prod_cf_vctr(j,1)-
(prod_cf_vctr(j,1)+total_deprtn_vcrt(j,1))*tax_rate;
  end
  investments(i,1)=(-1)*sum(capex_vctr);
end
for i=1:m
  for j=1:300
      if j==1
          Cum_Cash_flow(j,i)=Cash_flow(j,i);
```

```
        else
            Cum_Cash_flow(j,i)=Cum_Cash_flow(j-1,i)+Cash_flow(j,i);
        end
    end
end
NPV_data=0; NPV_data_succ=0;
PI_data=0; PI_data_succ=0;
FV_data=0; GRR_data_succ=0; GRR_data=0;
disc_rate=.12/12;
count=1;
for i=1:m
    NPV_data(i,1)=0; FV_data(i,1)=0; GRR_data(i,1)=0;
    PI_data(i,1)=0;
    for j=1:300
        if drill_succ(i,1)>0 && Cash_flow(j,i)>0
FV_data(i,1)=FV_data(i,1)+Cash_flow(j,i)*((1+disc_rate)^(300-j));
        elseif drill_succ(i,1)==0
            FV_data(i,1)=0;
        end
        NPV_data(i,1)=NPV_data(i,1)+Cash_flow(j,i)/((1+disc_rate)^j);
    end
    GRR_data(i,1) = (1+((FV_data(i,1)/investments(i,1))^(1/300)-1))^12-
1;
    PI_data(i,1) = 1+NPV_data(i,1)/investments(i,1);
    if drill_succ(i,1)>0
        PI_data_succ(count,1) = 1+NPV_data(i,1)/investments(i,1);
        NPV_data_succ(count,1) = NPV_data(i,1);
        GRR_data_succ(count,1) =
(1+((FV_data(i,1)/investments(i,1))^(1/300)-1))^12-1;
        count=count+1;
    end
end
acc_NPV_data = NPV_data;
acc_NPV_data_succ = NPV_data_succ;
acc_PI_data = PI_data;
acc_PI_data_succ = PI_data_succ;
```

```
acc_GRR_data = GRR_data;
acc_GRR_data_succ = GRR_data_succ;
```

MATLAB CODE FOR GATHERING DATA AND GENERATING

OPPORTUNITY SET AND EFFICIENT FRONTIER

```
NPV_data_total=0;
NPV_data_succ_total =0;
GRR_data_total =0;
GRR_data_succ_total =0;
PI_data_total =0;
PI_data_succ_total =0;


NPV_data_total = [acc_NPV_data acc_sol_NPV_data dld_NPV_data
dld_sol_NPV_data dbl_NPV_data dbl_sol_NPV_data cnt_NPV_data
cnt_sol_NPV_data tgh_NPV_data hrr_NPV_data lrr_NPV_data];
NPV_data_succ_total = [acc_NPV_data_succ acc_sol_NPV_data_succ
dld_NPV_data_succ dld_sol_NPV_data_succ dbl_NPV_data_succ
dbl_sol_NPV_data_succ cnt_NPV_data_succ cnt_sol_NPV_data_succ
tgh_NPV_data_succ hrr_NPV_data_succ lrr_NPV_data_succ];


GRR_data_total = [acc_GRR_data acc_sol_GRR_data dld_GRR_data
dld_sol_GRR_data dbl_GRR_data dbl_sol_GRR_data cnt_GRR_data
cnt_sol_GRR_data tgh_GRR_data hrr_GRR_data lrr_GRR_data];
GRR_data_succ_total = [acc_GRR_data_succ acc_sol_GRR_data_succ
dld_GRR_data_succ dld_sol_GRR_data_succ dbl_GRR_data_succ
dbl_sol_GRR_data_succ cnt_GRR_data_succ cnt_sol_GRR_data_succ
tgh_GRR_data_succ hrr_GRR_data_succ lrr_GRR_data_succ];


PI_data_total = [acc_PI_data acc_sol_PI_data dld_PI_data
dld_sol_PI_data dbl_PI_data dbl_sol_PI_data cnt_PI_data cnt_sol_PI_data
tgh_PI_data hrr_PI_data lrr_PI_data];
PI_data_succ_total = [acc_PI_data_succ acc_sol_PI_data_succ
dld_PI_data_succ dld_sol_PI_data_succ dbl_PI_data_succ
dbl_sol_PI_data_succ cnt_PI_data_succ cnt_sol_PI_data_succ
tgh_PI_data_succ hrr_PI_data_succ lrr_PI_data_succ];
```

```
for f=1:11

    NPV_data_means_vector(f,1)=mean(NPV_data_total(:,f));
    NPV_data_succ_means_vector(f,1)=mean(NPV_data_succ_total(:,f));
    GRR_data_means_vector(f,1)=mean(GRR_data_total(:,f));
    GRR_data_succ_means_vector(f,1)=mean(GRR_data_succ_total(:,f));
    PI_data_means_vector(f,1)=mean(PI_data_total(:,f));
    PI_data_succ_means_vector(f,1)=mean(PI_data_succ_total(:,f));
    NPV_data_stds_vector(f,1)=std(NPV_data_total(:,f));
    NPV_data_succ_stds_vector(f,1)=std(NPV_data_succ_total(:,f));
    GRR_data_stds_vector(f,1)=std(GRR_data_total(:,f));
    GRR_data_succ_stds_vector(f,1)=std(GRR_data_succ_total(:,f));
    PI_data_stds_vector(f,1)=std(PI_data_total(:,f));
    PI_data_succ_stds_vector(f,1)=std(PI_data_succ_total(:,f));
end


GRR_data_succ_cov = cov(GRR_data_succ_total);
PI_data_succ_cov = cov(PI_data_succ_total);
NPV_data_succ_cov = cov(NPV_data_succ_total);
GRR_data_cov = cov(GRR_data_total);
PI_data_cov = cov(PI_data_total);
NPV_data_cov = cov(NPV_data_total);


Cov_mtrx = NPV_data_succ_cov;
sds = NPV_data_succ_stds_vector;
ret = NPV_data_succ_means_vector;
n=10000;
prj_wghts_array=0;


for j=1:10000
nmbr_prj=11;
indexes(1:nmbr_prj,1)=1:nmbr_prj;
wght=0;
resid=1;
```

```
prj=0;
prj_wghts=0;


for i=1:nmbr_prj
prj=round(unifrnd(.51,(nmbr_prj+1.49-i),1,1));
wght=unifrnd(0,resid,1,1);
prj_wghts(i,1)=indexes(prj,1);


if i==nmbr_prj
    prj_wghts(i,2)=resid;
else
    prj_wghts(i,2)=wght;
end


indexes(prj,:)=[];
resid=resid-wght;
end


prj_wghts=sortrows(prj_wghts,1);
prj_wghts_array(1:nmbr_prj,j)=prj_wghts(:,2);
end


prj_wghts_array=prj_wghts_array';
for i=1:n
    Rp(i,1) = prj_wghts_array(i,:)*ret;
    SD_p(i,1) =
sqrt(prj_wghts_array(i,:)*Cov_mtrx*prj_wghts_array(i,:)');


end
plot(SD_p,Rp,'*');
efficient_frontier=0;
efficient_proj_weights_array=0;
efficient_proj_weights_array(1,nmbr_prj)=0;
z=1;
for j=1:10000
```

```
    cont=0;
    for v=1:10000
        if SD_p(v,1)<SD_p(j,1) && Rp(v,1)>=Rp(j,1)
            cont=1;
            break
        end
    end
    if cont==1
        continue
    end
    efficient_frontier(z,1)=SD_p(j,1); efficient_frontier(z,2)=Rp(j,1);
efficient_frontier(z,3)=3;
    efficient_proj_weights_array(z,:)=prj_wghts_array(j,:);
    z=z+1;
end

plot(efficient_frontier(:,1),efficient_frontier(:,2),'*');
xlswrite('efficient_frontier_NPV', [efficient_proj_weights_array
efficient_frontier]);
```

VITA

Name: Gizatulla Aibassov

Address: Texas A&M University
                          Attn: Dr. W.J. Lee
                          Harold Vance Department of Petroleum Engineering
                          3116 TAMU
                          College Station, TX 77843-3116

Email Address: aibasov@gmail.com

Education: M.S., Petroleum Engineering, 2007
                          Texas A&M University
                          College Station, Texas

                          B.Sc., Chemistry, 2004
                          al-Farabi Kazakh National University
                          Almaty, Kazakhstan